CONTAGION AND DECOUPLING IN INTERMEDIATED FINANCIAL MARKETS

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Abstract. I analyze the interplay between fundamental and intermediation risk in a multi-asset dynamic general equilibrium model with heterogeneous agents. Agents differ in their level of direct access to investment opportunities. Intermediation relationships are formed to overcome limited market access. Intermediation risk is captured via frictions in the relationships between agents that introduce fragility into asset prices. Asset prices are fragile when they have a concentrated investor base making them dependent on the fortunes of a few investors. In contrast, a non-concentrated investor base makes asset prices resilient with respect to intermediation risk. But not all assets with a concentrated investor base are fragile. I identify fundamental characteristics that induce resilience in assets with a common concentrated investor base. These characteristics lead to portfolio rebalancing within the common investor base that makes some assets resilient and renders others fragile in the presence of intermediation risk. Likewise, in a multi-asset framework, assets that are resilient due to a broad investor base are not completely immune to the fragility experienced by other assets. In a dynamic context, fragile assets tend to experience contagion whereas resilient assets tend to decouple whenever the intermediation frictions are severe. I argue that an understanding of the dynamic behavior of asset prices requires an understanding of fundamental and intermediation risk as well as the interaction between the two.

1. Introduction

A central insight in classical finance is that the price of an asset is determined by its fundamental cash-flow characteristics. What matters are the characteristics of its cash flows and what consumption opportunities these cash flows allow the holders to reach. If, for example, the current holder of an asset needs to trade for reasons unrelated to the assets fundamentals, some other investor will simply take his place without any impact on price. Yet, a growing literature documents that non-fundamental characteristics such as the structure of ownership or the institutional characteristics of markets do matter for asset prices. For example, Greenwood and Thesmar [2011] show that ownership concentration can pose a non-fundamental risk to prices. In their framework, assets can be fragile because they face non-fundamental demand shocks

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due to concentrated ownership. Apparently, some friction must exist such that a smooth replacement in the investor base is not possible. These frictions are a main concern in the literature on the limits of arbitrage that investigates the impediments arbitrageurs face when performing their work (e.g. Shleifer and Vishny [1997], Gromb and Vayanos [2002], Brunnermeier and Pedersen [2009]). Finally, there is evidence that characteristics of the market structure itself impact asset prices. Adrian et al. [2011] show that an asset pricing model with one factor that captures the leverage of security broker-dealers captures the cross-section of stock and treasury bond returns remarkably well. They motivate their research by recent asset pricing theory in which intermediaries play a central role in the market structure in which assets are traded (e.g. He and Krishnamurthy [2012b], Brunnermeier and Sannikov [2010]). In those models, frictions in the intermediation sector have price impact.

In this paper, I analyze the interplay between fundamental and intermediation risk in the cross-section of expected asset returns. I build a multi-asset dynamic general equilibrium model with intermediation between heterogeneous agents. In the model, some investors have only limited direct access to investment opportunities - they can invest directly only in some of the risky assets in the economy. These investors can overcome their impediment by forming relationships and co-investing with others. These relationships are, however, subject to frictions. The severity of the frictions captures a form of non-fundamental risk that I refer to as "intermediation risk". In a multi-asset framework, this intermediation risk interacts with the fundamental cash flow characteristics and generates interesting asset pricing patterns. The central point is that the behavior of asset prices cannot be understood by analyzing either fundamental or intermediation risk in isolation. This is for the following reasons. First, for as long as intermediation risk has not materialized, equilibria in the different model variations are independent of the level of direct access of the different investors. However, once intermediation risk materializes, the equivalence breaks down. This means that, in "good times", only fundamental risk is relevant for pricing assets whereas in "bad times", intermediation risk has price impact. As such, understanding the underlying market structure is central to understanding asset price responses in different states of the world. Second, once assets are simultaneously subject to intermediation risk, their different fundamental characteristics determine their price responses. I describe these price responses in terms of fragility or resilience in the static context, i.e. the extent to which prices respond to intermediation risk. In the dynamic context, I show that fragile assets tend to experience contagion while resilient assets tend decouple where contagion (decoupling) is defined as positive (negative) excess correlation relative to fundamental levels.

The model has three key ingredients. First, there are two types of agents that differ in their direct access to investment opportunities. Specialists can access all available investment opportunities, households only a few. Second, households can co-invest with specialists who then act as delegated portfolio managers in the resulting

intermediary relationships. Whenever households do so, they gain access to investment opportunities but become subject to the value-destructive behavior of money managers as in He and Krishnamurthy [2012b]. Once in charge of managing assets, specialists make an unobserved "work" or "shirk" decision. To induce "working", households require specialists to hold a minimum fraction of equity in the intermediary they manage, i.e. specialists need "skin in the game". Specialists, in turn, gain from such relationships by earning fee income. Intermediation risk materializes whenever specialists cannot contribute as much equity as is needed to support delegation demand from households. In those situations, delegation demand from households needs to be rationed. Third, the economy features multiple assets in positive net supply and the focus of the paper is on cross-asset implications when household delegation demand needs to be rationed and when the direct investment opportunities of households vary.

I begin with an equivalence result (proposition 1) that holds exactly in one-period examples and qualitatively in the full dynamic model. For as long as specialists are rich enough to support all delegation demand from households, equilibria are independent of the level of direct access households enjoy. This is because specialists compete against each other to earn fee income from selling their superior market access to households. This drives down the cost of intermediation provided specialist's wealth can support all delegation demand given the equity capital constraint. In those situations, the model delivers integrated markets: perfect risk sharing, common access to investment opportunities, efficient allocations. In the language of asset pricing, markets are complete as everybody faces the same market price of risk. Risk premia are determined in the standard fashion by the co-variation of individual cash flows with aggregate consumption.

This equivalence breaks down once the capital constraint binds. Such situations are labeled "crises". Crises are understood as a materialization of intermediation risk. They happen when specialists are poor relative to households. The second result is that overall price fragility in the economy is higher, the more limited the direct access of households. In the cross-section of assets, prices of assets that can be held directly by both households and specialists are resilient whereas prices of assets that can be held by specialists only tend to be fragile (proposition 2). This result is a direct consequence of the level of risk sharing the economy allows. When households rely heavily on the intermediation services of specialists, any disruption in delegation leads to sub-optimal savings and consumption decisions. When households delegation demand cannot be met, they retreat heavily into the risk-free bond market. In turn, specialists borrow to clear markets and are forced to lever up at times when they are reluctant to do so because of diminished wealth. At the same time, fundamental risk is concentrated among specialists. This amplifies intermediation risk. The forced leverage of specialists introduces a structural break in the stochastic discount factor that affects all assets. In the model, the market prices of risk start diverging. The severity of the incompleteness is relaxed when households enjoy more direct access to risky investment opportunities.

I demonstrate this structural break in multiple ways by varying either the level of direct access that households enjoy or the fundamental cash flow characteristics of the assets in the economy. First, I show that the more limited the direct access of household, the larger the drop in the risk-free rate. Second, in the static context, risk premia of assets that can only be held by specialists increase heavily (they are fragile) while risk premia of assets that enjoy direct access by households are hardly affected (they are resilient) and may, in special cases, even fall. In the dynamic context, I demonstrate that, during crises, assets that are both fragile due to a common, non-replaceable investor base tend to experience contagion while assets whose investor base can be replaced, tend to experience decoupling (proposition 4). Decoupling in such cases is not only due to a replacement of the investor base. Imperfect risk sharing forces specialists to concentrate their portfolios in few assets. Ideally, they would like to diversify but the intermediation frictions prevent first-best diversification. To lower specialists demand for resilient assets, their risk premia tend to (marginally) fall contributing to resilience.

These results seem to suggest that only the level of direct access a given asset enjoys determines its fragility or resilience and the level of contagion or decoupling it experiences with other assets during crises. In propositions 3 and 5, I show that this is not necessarily the case. I use the multi-asset framework to show that when multiple assets can be held by specialists only, their crisis behavior depends on their level of fundamental risk. I identify payoff characteristics conducive of resilience and decoupling during crises. These include: a low fundamental cash flow correlation with other assets, a small size of cash flows and a skewed distribution of cash flows over future states. As is standard in consumption based asset-pricing, when an asset has a high fundamental correlation with aggregate dividends, it is risky as it provides little diversification benefit. Holding its fundamental correlation constant, if the asset is small relative to the aggregate, its contribution to aggregate consumption is small which makes it less risky. Finally, the distribution of cash flows over future states also matters. Assets with non-symmetric or skewed payoffs can decouple from other assets with the same investor base because their future cash flow streams provide insurance properties that are more desirable in crisis periods. Assets with such characteristics tend to decouple because of a substitution effect. When their common investor base is subject to an adverse shock, all investors would like to rebalance towards the small (almost "risk-free") assets. In general equilibrium, this is not possible. Hence the risk premia of large assets have to increase and risk-premia of small assets have to decrease. Decoupling occurs. I illustrate how the substitution effect can produce negative risk premia during crises or, for example, an inverted term structure.

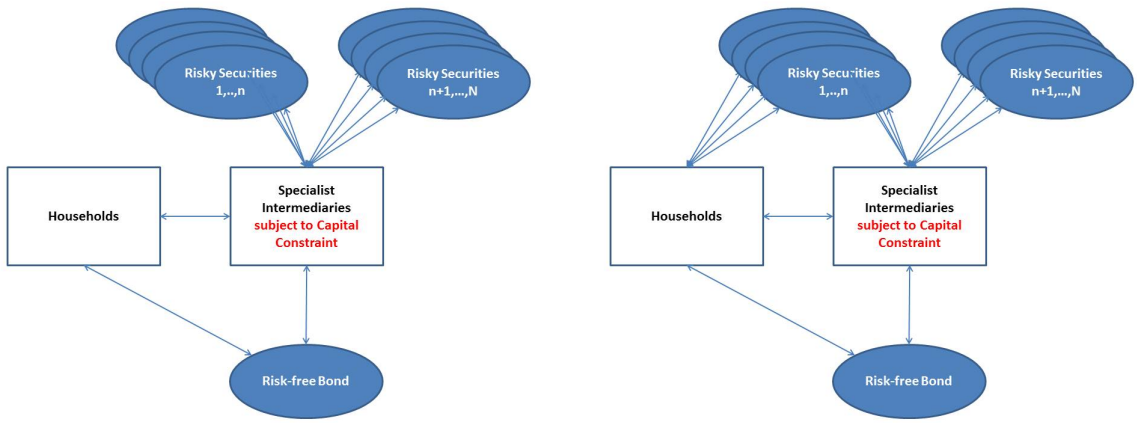
While I focus on a model with two investor populations, my framework is sufficiently general to accommodate multiple types of heterogeneous agents with iso-elastic utility and multiple risky assets in positive

supply. I model the intermediation game between households and specialists in one system. Both households and specialists optimize their equity contribution to the intermediary. Because households wish to implement "working" by the specialists, they self-impose an incentive compatibility constraint on their optimization problem. I show how this formulation mimics a competitive market in intermediation services with endogenous transfers ("fees") from households to specialists. These transfers disturb the market prices of risk that different agents face and therefore determine whether markets are complete or incomplete as a function of aggregate states.

I contribute to various strands of the literature. The first ingredient of the model is limited market participation on the part of households. Limited market participation was previously shown useful in explaining various asset pricing anomalies. Mankiw and Zeldes [1991] as well as Basak and Cuoco [1998] show that limited stock market participation can go a long way in explaining the equity premium puzzle. Vissing-Jorgensen [2002] shows that accounting for limited asset market participation is important when estimating the elasticity of intertemporal substitution. Guvenen [2009] emphasizes the link between the two. Allen and Gale [1994] study the impact of limited participation on volatility.

The second ingredient is intermediation with frictions. These frictions lead to wealth effects that affect asset prices. The impact of wealth effects on multiple assets is the subject of Kyle and Xiong [2001]. Gromb and Vayanos [2009] also study the cross-market implications of constrained arbitrageurs. From an asset pricing perspective, the capital constraint that households impose on specialists because of moral hazard and its impact on asset prices relates this paper to the literature on the equilibrium impact of portfolio constraints (e.g. Basak and Croitoru [2000], Pavlova and Rigobon [2008]). The difference to these models is that I focus on a constraint on the capital structure of intermediaries rather than on a constraint on individual portfolio decisions. In addition, the Lagrange multipliers that are associated with the capital structure constraint turn out to be prices in a parallel market for intermediation services. As such, they are internalized in the equilibrium because they determine transfers between investors and feed back into decisions via the budget constraints. Also, my model does not feature differentiated goods. There exist multiple trees that generate the same consumption good and goods market are frictionless. The asset pricing phenomena that are associated with that structure are the subject of Cochrane et al. [2008] and Martin [Forthcoming].

Closest to this paper are the papers of He and Krishnamurthy [2012b,a]. I borrow the intermediation framework from those papers and add two significant extensions. First, I focus on the case of multiple assets in positive supply and show that the model produces rich asset pricing patterns depending on the fundamental characteristics of the assets in the economy. Second, I allow households to access some assets directly which makes them less dependent on specialists. This appears a realistic extension as we observe direct access



"Segmentation" Economy "Partial Access" Economy

FIGURE 1. **The Economies**

by retail investors in at least some assets. I show that this extension has strong equilibrium implications. Further, I provide a framework that, in principle, allows for more than two investor populations. This opens avenues for future research that can model even richer interactions with contracting between multiple investor populations.

The paper proceeds as follows. In section 2, I present the modeling framework and one period versions of two configurations that I solve analytically. The two configurations are illustrated in figure 1. I consider the cases when households have no direct access to risky asset investment ("Segmentation" economy) or when they can access a subset of risky assets ("Partial Access" economy). I use the static analysis to derive conditions for the fragility or resilience of asset prices as a function of endogenous states. In section 3, I analyze equilibria in the dynamic counterparts to the configurations studied before. Since markets are incomplete, the equations that characterize equilibria are difficult to solve for analytically. I resort to numerical techniques and show that situations in which risk premia of two assets were fragile in the static analysis translate into contagion whereas situations in which risk premia in were resilient translate into decoupling. The configurations I discuss in sections 2 and 3 have two classes of agents but the framework is capable of accommodating a larger number of heterogeneous investor populations. Therefore, in section 4, I discuss such a general framework in which multiple agent populations contract with each other and outline the avenues for future research that such a general framework opens. I conclude in section 5.

2. THE STATIC CASE

2.1. **Set Up.** Time is discrete and indexed by $t = 0, ..., T$. In the static versions I describe here, there are only three periods: $t = T-2$, $T-1$, T. The period $T-2$ indexes "initial endowments" in terms of portfolio holdings with which agents start their lives. At $T-1$, agents choose portfolios and consumption for the

first time and the economy ends with a terminal period T in which agents consume only. The economy is populated by different classes of agents (each in unit mass) that differ in the degree of direct access to investment opportunities. Specialists can directly access every investment opportunity, households can only access a subset of all existing investment opportunities but can hire specialists to invest on their behalf. These intermediation relationships are described shortly.

There is one consumption good in the economy and it is produced by N Lucas [1978]-trees indexed by $n = 1, ..., N$ that generate dividends according to the processes

$$\delta_{n,T} = g_n \times \delta_{n,T-1} + \epsilon_{n,T} \tag{2.1}$$

where g_n is the tree-specific expected dividend growth and $\epsilon_{n,T}$ are shocks to the terminal dividends. The $\epsilon_{n,T}$-shocks have zero mean and variance σ_n^2 and may be correlated across trees. Every tree n is represented by an asset with price $P_{n,T-1}$. A zero net-supply risk-free bond that pays one unit of the consumption good in the terminal period exists, its price is B_{T-1}. Both households and specialists maximize expected utility of the form

$$\mathbb{E}_{T-1} \sum_{\tau=0}^{1} U_i \left(c_{i,t+\tau} \right) \; i \in \{S, H\} \tag{2.2}$$

The set-up I describe here is general in the sense that it spans all levels of limited access that households may have. The configurations in figure 1 are but two that are chosen to illustrate the main mechanisms at work. The set-up encompasses both the "segmentation" economy where households have no direct access to risky assets (and of which He and Krishnamurthy [2012b] is a sub-case when $N = 1$, utility is of the log form and there are multiple periods) and the "partial access" economy where households can access some risky assets directly.

Since goods market are frictionless, households would generally like to access every investment opportunity to have an optimally diversified portfolio that finances their final consumption which is subject to all $\epsilon_{n,T}$-shocks. This creates a need for intermediation as there are gains from trade from hiring a specialist to provide such risky asset exposure. To hire specialists, households can access a market for intermediation services in which specialists compete to form relationships with households.

When households hire a specialist and co-invest with him, they buy access to unreachable investment opportunities but become subject to his value destructive behavior. Households delegate funds to specialists. Once in charge, specialists make an unobserved due diligence decision of either "working" ($s_{T-1} = 0$) or "shirking" ($s_{T-1} = 1$). "Shirking" creates a dollar cost of X_{T-1} that is borne by all parties in the intermediation relationship but it also generates a private benefit of Z_{T-1} that accrues to the specialist only,

with $X_{T-1} > Z_{T-1} > 0$. To alleviate moral hazard, households can write linear share / fixed fee contracts that define a sharing rule for the returns of the intermediary as well as a transfer between households and specialists upon initiation of the intermediary as in He and Krishnamurthy [2012b]. These authors show that, without loss of generality, such contracts are described by the contract terms $\{\theta_{S,I,T-1}, F_{T-1}\}$ where $\theta_{S,I,T-1} \in [0,1]$ is the sharing rule and F_{T-1} is the transfer (a "fee"). The sharing rule $\theta_{S,I,T-1}$ defines how the value of intermediary I that is managed by specialist S is shared. Intermediaries are a form of co-investment vehicle and $\theta_{S,I,T-1}$ is the claim to its equity. As such, it can be treated as an additional portfolio choice variable. To avoid any unwinding of incentives set by the contract terms, specialists cannot have outside risky asset exposure. Whatever wealth they do not consume or contribute to intermediaries earns the risk-free rate. The size of the transfer F_{T-1} is the outcome in a competitive intermediation market and described shortly.

To keep the focus on the main moral hazard friction, I assume that only the "work" or "shirk" decision is unobservable, the portfolio compositions that specialists subsequently choose on behalf of households are observable. This distinction is necessary because it impacts the way the budget constraints are written for both households and specialists. He and Krishnamurthy [2012b] (Lemma 4) show that from the households' perspective, the transfer to specialists is proportional to the risk exposure households obtain. When the portfolio choice is observable, households pay specialists as a direct function of the sharing rule $\theta_{S,I,T-1}$. This means that specialists take transfers into account when choosing how much risk exposure to provide and how to choose the composition of the intermediary portfolio.[1]

With such contracts in mind, the current and future budget constraints of a representative specialist become

$$
\begin{aligned}
c_{S,T-1} + \theta_{S,I,T-1}\left(\sum_{n=1}^{N} \alpha_{n,T-1} P_{n,T-1} + s_{T-1} X_{T-1}\right) + \theta_{S,B,T-1} B_{T-1} & \\
- \left(1 - \theta_{S,I,T-1}\right) F_{T-1} - s_{T-1} Z_{T-1} &= \\
\theta_{S,I,T-2}\left(\sum_{n=1}^{N} \alpha_{n,T-2}\left(P_{n,T-1} + \delta_{n,T-1}\right)\right) + \theta_{S,B,T-2} & \\
c_{S,T} = \theta_{S,I,T-1}\left(\sum_{n=1}^{N} \alpha_{n,T-1} \delta_{n,T}\right) + \theta_{S,B,T-1} &
\end{aligned}
\tag{2.3}
$$

[1] See their Appendix A.7 for details. The difference is that with unobservable portfolio choice inside the intermediary, households pay specialists based on expected exposure which they need to infer from specialist's wealth. With rational expectations, this expected exposure coincides with true exposure. But for specialists, this means that their choice of exposure supply is not affected by "fee income". When intermediary portfolios are observable, specialists take fees into account in their choice of exposure supply. This would mean that in budget constraints 2.3 and 2.5, the transfer F_{T-1} would not be pre-multiplied by the exposure households receive, $1 - \theta_{S,I,T-1}$.

where c_S denotes consumption of specialist S, $\theta_{S,I}$ the sharing rules of intermediary I managed by specialist S, $\{\alpha_n\}$ the compositions of the intermediary portfolios chosen by the specialists that can include all $n \in N$, $\theta_{S,B}$ the holdings in the risk-free bond B by specialist S and F the transfers from households. "Initial endowments" are represented by the portfolio shares with time index $T-2$. Agents derive utility from consumption only. Therefore, all prices collapse to zero in the final period which is reflected in the terminal budget constraint (fourth row above). The right-hand-side in the budget constraints represent the entering wealth that reflects decisions taken in the previous period. They finance today's choices that include the "work" or "shirk" decision. If a specialist chooses to "shirk" ($s_{T-1} = 1$), he obtains a private benefit Z_{T-1} that relaxes his budget constraint but he suffers a cost of $\theta_{S,I,T-1}X_{T-1}$ that tightens the constraint. By engaging with households, he can earn fee income proportional to the exposure he provides which also relaxes his budget constraint.

Specialists will choose to "work" or "shirk" depending on the sharing rule $\theta_{S,I,T-1}$. The terms on the left-hand-side of equation 2.3 that are affected by the due diligence decision s_{T-1}, are $s_{T-1}\left(\theta_{S,I,tT-1}X_{T-1} - Z_t\right)$. Households need to insist on a sharing rule that makes "shirking" unattractive to specialists if they want to implement $s_{T-1} = 0$, i.e. a sharing rule that obeys an incentive compatibility constraint

$$\begin{aligned} \theta_{S,I,T-1}X_{T-1} - Z_{T-1} &\geq 0 \\ \beta = \frac{Z_{T-1}}{X_{T-1}} &\leq \theta_{S,I,T-1} \end{aligned} \tag{2.4}$$

The specialists needs to have "skin in the game" in order to be properly incentivized to work.

For households, the budget constraints are defined accordingly. Let $\hat{N} \subset N$ denote the subset of risky assets to which households have direct access. They are ordered in increasing fashion, i.e. households have direct access to assets $n = 1, ..., \hat{N}$ while specialists have access to assets $n = 1, ..., \hat{N}, ...N$.

$$\begin{aligned} c_{H,T-1} + (1-\theta_{S,I,T-1})\left(\sum_{n=1}^{N}\alpha_{n,T-1}P_{n,T-1} + s_{T-1}X_{T-1} + F_{T-1}\right) \\ + \sum_{n=1}^{\hat{N}}\hat{\alpha}_{n,T-1}P_{n,T-1} + \theta_{H,B,T-1}B_{T-1} = \\ (1-\theta_{S,I,T-2})\left(\sum_{n=1}^{N}\alpha_{n,T-2}\left(P_{n,T-1}+\delta_{n,T-1}\right)\right) + \sum_{n=1}^{\hat{N}}\hat{\alpha}_{n,T-2}\left(P_{n,T-1}+\delta_{n,T-1}\right) + \theta_{H,B,T-2} \\ c_{H,T} = (1-\theta_{S,I,T-1})\left(\sum_{n=1}^{N}\alpha_{n,T-1}\delta_{n,T-1}\right) + \sum_{n=1}^{\hat{N}}\hat{\alpha}_{n,T-1}\delta_{n,T-1} + \theta_{H,B,T-1} \end{aligned} \tag{2.5}$$

The budget constraints reflect the gains from trade for both parties. Households obtain access to otherwise unreachable investment opportunities but become subject to the value destructive behavior of specialists.

"Shirking" on the part of specialists tightens the budget constraint of households as it constitutes an additional cost without associated benefits. Specialists, in turn, can monetize their superior market access and earn via transfers from households.

Let me assume that households always wish to implement $s_{T-1} = 0$. I give a sufficient condition such that this is indeed optimal in lemma 1. The $T-1$ Lagrangians for specialists are stated in equation A.1 in the Appendix (ϕ_S denote the multipliers on the budget constraints). The associated optimality conditions are

$$
\begin{aligned}
U_S'\left(c_{S,T-1}\right) &= \phi_{S,T-1} \\
\mathbb{E}_{T-1}\left[\phi_{S,T}\left(\sum_{n=1}^{N}\alpha_{n,T-1}\delta_{n,T}\right)\right] &= \phi_{S,T-1}\left(\sum_{n=1}^{N}\alpha_{n,T-1}P_{n,T-1}+F_{T-1}\right) \\
\mathbb{E}_{T-1}\left[\phi_{S,T}\delta_{n,T}\right] &= \phi_{S,T-1}P_{n,T-1}\,\forall n\in N \\
\mathbb{E}_{T-1}\left[\phi_{S,T}\right] &= \phi_{S,T-1}B_{T-1} \\
& B.C.
\end{aligned}
\tag{2.6}
$$

Specialists optimize with respect to their consumption (first row in equation 2.6), contribution to the intermediary (second row in equation 2.6), the portfolio compositions in the N risky assets (third row in equation 2.6) and risk-free savings (fourth row in equation 2.6), taking as given the market price of intermediation as well as security prices.

The corresponding household problem is similar with the difference that households wish to implement working, hence the amount they delegate needs to be incentive compatible for the specialists, given the specialists optimal choice of equity contribution. Therefore, the household problems needs to obey equation 2.4 which adds an inequality constraint to their problems. In addition, households have no direct influence over the portfolio composition of the intermediary they invest in, they only participate in the specialists' portfolio. In total, households optimize with respect to consumption, contribution to intermediaries while respecting the capital constraints, risk-free savings and direct risky investment in the subset $\hat{N}$ of assets they can access themselves. Their $T-1$ Lagrangians are stated in equation A.2 in the Appendix. The associated optimality conditions, where ϕ_H denote the multipliers on the budget constraints and ϕ^{CC} the multiplier on the capital constraints, are

$$
\begin{aligned}
U_H'\left(c_{H,T-1}\right) &= \phi_{H,T-1} \\
\mathbb{E}_{T-1}\left[\phi_{H,T}\left(\sum_{n=1}^{N}\alpha_{n,T-1}\delta_{n,T}\right)\right] &= \phi_{H,T-1}\left(\sum_{n=1}^{N}\alpha_{n,T-1}P_{n,T-1}+F_{T-1}\right)+\phi_{T-1}^{CC} \\
\mathbb{E}_{T-1}\left[\phi_{H,T}\delta_{n,T}\right] &= \phi_{H,T-1}P_{n,T-1}\,\forall n\in\hat{N} \\
\mathbb{E}_{T-1}\left[\phi_{H,T}\right] &= \phi_{H,T-1}B_{T-1} \qquad (2.7) \\
\phi_{T-1}^{CC}\left(\beta-\theta_{S,I,T-1}\right) &= 0 \\
\theta_{S,I,T-1} &\geq \beta \\
& B.C.
\end{aligned}
$$

Equations 2.6 and 2.7 deliver a set of insights. Marginal utility of specialists prices any risky asset while marginal utility of households only prices those risky assets to which they have direct access. This means that specialists and households only agree on those prices to which they enjoy common access. This includes the risky assets to which households have access, the bond and the total value of the intermediary including transfers on which they both contract. For example, the first order conditions on the bond exposure delivers $\mathbb{E}_{T-1}\left[\frac{\phi_{H,T}}{\phi_{H,T-1}}\right]=\mathbb{E}_{T-1}\left[\frac{\phi_{S,T}}{\phi_{S,T-1}}\right]$. A similar condition obtains for price agreement on the value of the intermediary portfolio including transfers with the modification that it is distorted by the Lagrange multiplier on the capital constraint ϕ_{T-1}^{CC}. The condition is

$$
\mathbb{E}_{T-1}\left[\frac{\phi_{H,T}}{\phi_{H,T-1}}\left(\sum_{n=1}^{N}\alpha_{n,T-1}\delta_{n,T}\right)\right]-\frac{\phi_{T-1}^{CC}}{\phi_{H,T-1}}=\mathbb{E}_{T-1}\left[\frac{\phi_{S,T}}{\phi_{S,T-1}}\left(\sum_{n=1}^{N}\alpha_{n,T-1}\delta_{n,T}\right)\right] \tag{2.8}
$$

For as long as the incentive compatibility constraint is slack $\left(\phi_{T-1}^{CC}=0\right)$, households and specialists agree on the value of the intermediary portfolio. When the constraint starts binding, the desire of households to implement "working" drives a wedge into the price agreement condition. This wedge is intimately linked to the contractual transfer F_{T-1} and captures how the market prices of risk diverge between households and specialists. With a competitive intermediation market, the transfer from households to specialists is competed away for as long as specialists demand a sufficiently high sharing rule $\theta_{S,I,T-1}$ on their own. This is because intermediation supply is perfectly elastic for as long as the capital constraint is slack but becomes inelastic once it binds. Intermediation demand, however, is downward-sloping. So the transfer rises above zero once the capital constraint binds. To what level? Competitive pressure in the intermediation market will push transfers to the minimum level subject to satisfying the capital constraint. This minimum level needs to be such that households are no longer willing to delegate at the margin. Their willingness to delegate

is intimately captured in the Lagrange multiplier ϕ^{CC}_{T-1}, or the marginal utility of delegating at the optimal point. In other words, the demand curve at the optimal point once converted back into monetary amounts. To convert its value from utils into units of the consumption good, the equilibrium transfer F_{T-1} will turn out to be a function of the two multipliers ϕ^{CC}_{T-1t} and $\phi_{H,T-1}$, specifically $F_{T-1} = \frac{\phi^{CC}_{T-1}}{\phi_{H,T-1}}$ (see lemma 1 and its proof for further details).

2.2. **Definition of Equilibrium.** Equilibrium in the economy requires equilibrium in the market for intermediation as well as equilibrium in the capital and goods market.

Definition 1. Equilibrium in the intermediation market is described by the contract terms $\{\theta_{S,I,T-1},\, F_{T-1}\}$ where $\theta_{S,I,T-1}$ is incentive compatible for specialists. In equilibrium, there is no coalition of specialists and households with alternative contract terms such that households are strictly better off and specialists are weakly better off.

Next the capital and goods market equilibrium.

Definition 2. Capital and goods market equilibrium is defined as a set of $T-1$ prices $P_n\, \forall n \in N$ and B and portfolio policies $\{\theta_{S,I},\, \theta_{H,I},\, \theta_{S,B},\, \theta_{H,B},\, \{\alpha_n\},\, \{\hat{\alpha}_n\}\}$ as well as $T-1$ and T consumption policies $\{c_S,\, c_H\}$ and such that optimality conditions 2.6 and 2.7 hold, goods market clear, i.e. $\sum_{i\in\{S,H\}} c_{i,t} = \sum_{n\in N} \delta_{n,t}\, \forall t$ and capital markets clear, i.e. $\alpha_{n,T-1} + \hat{\alpha}_{n,T-1} = 1\, \forall n \in \hat{N}$, $\alpha_{n,T-1} = 1\, \forall n \in N \setminus \hat{N}$, $\theta_{S,B,T-1} + \theta_{H,B,T-1} = 0$ and $\theta_{S,I,T-1} + \theta_{H,I,T-1} = 1$.

2.3. **Intermediation Equilibrium.** I begin by analyzing equilibrium in the market for intermediation and directly state the equilibrium result.

Lemma 1. *<u>Intermediation Equilibrium</u> The intermediation equilibrium is symmetric, all households obtain the same exposure to risky assets and all specialists receive the same transfer. The contract terms are incentive compatible and satisfy $\theta_{S,I,T-1} \geq \beta$ and $F_{T-1} = \frac{\phi^{CC}_{T-1}}{\phi_{H,T-1}}$. For $X_{T-1} \geq \bar{X}_{T-1}$, where $\bar{X}_{T-1}$ is specified in equation B.3, households always wish to implement $s_{T-1} = 0$.*

Proof. See Appendix B. □

The main insight from lemma 1 is that the market price of intermediation can be inferred from the Lagrange multipliers of the individual household problem by recognizing that the ratio $\frac{\phi^{CC}_{T-1}}{\phi_{H,T-1}}$ is the free term that equates intermediation demand with intermediation supply - hence it needs to be the market price for intermediation services. In that sense, the competitive market result follows from individual optimization. This is similar compared to the literature of asset pricing with portfolio constraints where the Lagrange

multipliers on the portfolio constraints also disturb the individual market prices of risk such that the constraints are obeyed. The difference here is that the multipliers are real prices, rather than "shadow prices", in the separate market for intermediation services and that the constraint is imposed on the capital structure of the intermediaries rather than on direct portfolio holdings. In addition, the multipliers determine equilibrium transfers from households to specialists and therefore feed back into their decisions via the budget constraints.

2.4. **Asset Market Equilibrium.** I now turn to equilibrium in the capital and goods market of the static economies. One issue that needs to be clarified is the choice of state variables. Naturally, the individual dividends are exogenous state variables that drive the economy but the capital constraint dictates that specialists need to maintain a minimum stake in the intermediaries they manage. I will solve the model with iso-elastic utility, in which case the risks agents take depend on their wealth. Hence, an endogenous state variable to track the evolution of relative wealth is needed to determine whether the capital constraint 2.4 binds or not. There is some flexibility in picking the state variable. In the current formulation, the initial endowments in terms of portfolio holdings are the closest candidates. However, they are impractical as they are multi-dimensional.

For the purpose of solving the static models, I pick the current consumption share of specialists at time $T-1$ as the endogenous state variable. It is denoted $\omega = \frac{c_{S,T-1}}{c_{S,T-1}+c_{H,T-1}} = \frac{c_{S,T-1}}{\sum_{n=1}^{N}\delta_{n,T-1}}$. The current consumption share is a convenient choice. First, there exists a mapping from the multi-dimensional initial portfolio endowments to current consumption just as there exists mapping from the initial endowments to initial wealth. Second, since consumption cannot turn negative and is tied to aggregate dividends via goods market clearing, it has a clearly defined range. This will prove useful in the next section. Third, it turns out that the solutions are tractable using the current consumption shares of specialists as endogenous state variable.

With the choice of endogenous state variable ω, I am left to solve for the optimal consumption in all terminal states, the portfolios chosen today to finance these terminal consumption plans as well as prices. I solve under the parametrization that all agents have CRRA utility of the form $U(c_{i,t}) = e^{-\rho_i t}\frac{c_{i,t}^{1-\gamma_i}}{1-\gamma_i}$ where ρ_i is the time preference parameter and γ_i governs risk aversion. In this section, I restrict the analysis to the case with equal risk aversion ($\gamma_H = \gamma_S = \gamma$). The other exogenous parameters include all current and future dividends, the severity of moral hazard captured by the tightness of the capital constraint via the parameter $\beta_{T-1} = \frac{Z_{T-1}}{X_{T-1}}$ and the time preference rate, where I assume $\rho_H > \rho_S$ as in He and Krishnamurthy [2012b]. The risk-free rate is denoted $1 + rf_{T-1} = \frac{1}{B_{T-1}}$, the risk premia for risky assets are denoted $\mu_{n,T-1}$, $n \in N$

and defined multiplicatively as $1+\mu_{n,T-1} = \left(\frac{\mathbb{E}_{T-1}[\delta_{n,\eta,T}]}{P_{n,T-1}}/(1+rf_{T-1})\right)$, η denotes future states over which expectations are taken.

The main mechanisms of the model are understood by focusing on examples with two risky assets where the direct access of households to one of them varies. In the first configuration, labeled the "segmentation" economy (left panel in figure 1), households have no direct access, i.e. $\hat{N} = \emptyset$. In the second, labeled the "partial access" economy (right panel in figure 1), households have access to one risky asset, i.e. $\hat{N} = \{1\}$. Whenever it is instructional, I plot the results. In those cases, I use baseline parameters that are summarized in figure 2. Unless otherwise stated, the risky assets are calibrated to be exactly identical in terms of their

Parameter	Value
Severity of Moral Hazard	$\beta = 20\%$
Time Preference Rates	$\rho_S = 1.67\%$ $\rho_H = 4\%$
Dividend Growth Rates	$g_1 = g_2 = 1.7\%$
Dividend Volatility	$\sigma_1 = \sigma_2 = 10\%$
Dividend Correlation	$20\% \leq \rho_{12} \leq 80\%$
Risk Aversion	$1 \leq \gamma \leq 4$
Scale Factor on 2nd Asset Tree	$1/100 \leq s \leq 5$

FIGURE 2. **Baseline Parameter Choices**

expected dividend growth, volatility as well as higher moments. Section 3.1 gives more details on the tree calibration. At this stage, it is sufficient to know that an exogenous structure with two risky assets and one bond can be replicated on an event tree with three outgoing branches (i.e. future states) from every current node. In general equilibrium, this is crucial as any other specification with a different number of outgoing branches would change the completeness of the market.

The first proposition states the extent to which equilibria are equivalent in those configurations.

Proposition 1. *Equivalence When the capital constraint is slack, equilibria are independent of the degree of direct market access of households.*

Proof. See Appendices C and D. □

The main insight of proposition 1 is that, with equal risk aversion and only one final period, direct market access of households does not matter for as long as specialists are sufficiently rich. Direct access does not add value to households in good times because they can obtain their optimal risk exposure by delegating at zero cost due to perfect competition in the market for intermediation. This is exemplified by the sharing rule $\theta_{S,I,T-1}$ that is independent of $\hat{N}$. It is

$$\theta_{S,I,T-1} = Max\left[\beta,\, \omega\left(\left(\frac{e^{-\rho_H}}{e^{-\rho_S}}\right)^{\frac{1}{\gamma}}(1-\omega)+\omega\right)^{-1}\right] \tag{2.9}$$

When the capital constraint is slack (second element in the $Max\,[]$ function), total risk exposure is shared proportionally to the current consumption share of specialists. In addition, the solution steps and results in Appendices C and D show that bond market are dormant, i.e. $\theta_{S,B,T-1} = \theta_{H,B,T-1} = 0$, and that households optimally forgo to obtain direct risky asset exposure, i.e. $\hat{\alpha}_{1,T-1} = 0$, for as long as the capital constraint is slack.

The equivalence breaks down once the capital constraint binds. Such situations are labeled "crises" and are interpreted as a materialization of intermediation risk. The threshold when this occurs is again independent of $\hat{N}$. It is

$$\omega < \left(\frac{e^{-\rho_H}}{e^{-\rho_S}}\right)^{\frac{1}{\gamma}} \beta \left(1 + \beta \left(\left(\frac{e^{-\rho_H}}{e^{-\rho_S}}\right)^{\frac{1}{\gamma}} - 1\right)\right)^{-1} = \hat{\omega} \tag{2.10}$$

Whenever $\omega < \hat{\omega}$, the economies are constrained. Now, the degree of market access has strong implication for fragility or resilience of prices, both for risky assets and the risk-free bond. Fragile securities respond heavily to intermediation risk whereas the prices of resilient securities are little affected. The next two propositions discuss fragility and resilience of asset prices first with respect to direct market access of households and second with respect to fundamental payoff characteristics, holding direct market access constant.

Proposition 2. *Fragility and Resilience Prices are overall more fragile during crises the more restricted households direct market access is. Prices for assets household cannot access are fragile during crises while prices for assets with household access are resilient.*

Proposition 2 is a statement both about general asset price fragility as a function of direct market access and about the cross-section of asset price fragility and resilience when households access some risky assets but not others. I demonstrate asset price fragility by computing risk-free rates and risk premia for the two configurations, i.e. $\hat{N} = \emptyset$ or $\hat{N} = \{1\}$. Details of those calculations can be found in Appendices C.3 and D.3.

Start with the risk-free rate. At time $T-1$ it is

(2.11)

$$1 + rf(\omega) = \begin{cases} \left(e^{-\rho_S}\left(\sum_{n\in N}\delta_n\right)^{\gamma}\left(\left(\frac{e^{-\rho_H}}{e^{-\rho_S}}\right)^{\frac{1}{\gamma}}(1-\omega)+\omega\right)^{\gamma}\mathbb{E}\left[\left(\sum_{n\in N}\delta_{n,\eta}\right)^{-\gamma}\right]\right)^{-1} & if\,\omega \geq \hat{\omega} \\ \left(e^{-\rho_S}\left(\sum_{n\in N}\delta_n\right)^{\gamma}\omega^{\gamma}\mathbb{E}\left[\left(\beta\sum_{n\in N}\delta_{n,\eta} - \theta_{H,B}\right)^{-\gamma}\right]\right)^{-1} & if\,\omega < \hat{\omega}\,and\,\hat{N} = \emptyset \\ \left(e^{-\rho_S}\left(\sum_{n\in N}\delta_n\right)^{\gamma}\omega^{\gamma}\mathbb{E}\left[\left(\beta\left(\alpha_1\delta_{1,\eta} + \sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right) - \theta_{H,B}\right)^{-\gamma}\right]\right)^{-1} & if\,\omega < \hat{\omega}\,and\,\hat{N} = \{1\} \end{cases}$$

where I have dropped time subscripts.[2]

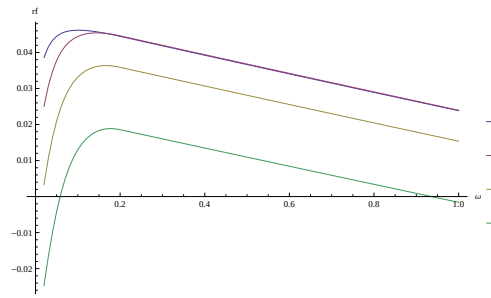
[2]Future dividends are identified by the subscript η that identifies future states. All other variables are either parameters or as of $T-1$.

In line with proposition 1, the risk-free rate is independent of $\hat{N}$ when the economy is unconstrained (first row in equation 2.11) but depends on $\hat{N}$ when the economy is constrained (second versus third row in equation 2.11) via the portfolio choice variable $\alpha_1 < 1$ (defined as an inverse function in the Appendix). In the unconstrained case, it is impacted by the current distribution of consumption only because the rates of impatience are different which leads to a slope negative in ω (first row above). If both households and specialists were equally impatient, the term featuring ω would collapse to 1 and the risk-free rate were flat. In figure 3, I plot the risk free rate and comparative statics for the baseline parameter values over the endogenous state variable ω for both the "segmentation" economy (panel A) and the "partial access" economy (panel B). In the unconstrained region, its comparative statics are standard. The risk-free rate decreases in risk aversion γ as well as in the fundamental correlation of the two risky assets ρ. Higher risk aversion (first graph in both panels) is associated with a higher precautionary motive, lowering the risk-free rate. A higher fundamental correlation (second graph in both panels) between dividends makes the aggregate consumption stream riskier lowering again the risk-free rate. The third graph in both panels varies the size of the second risky asset by scaling the initial size of the tree. When both trees are of equal size, the risk-free rate is highest. It falls when the trees are of unequal size. As in Martin [Forthcoming], an "unbalanced" economy is riskier since a higher fraction of aggregate consumption depends on one source of uncertainty which lowers the risk-free rate.

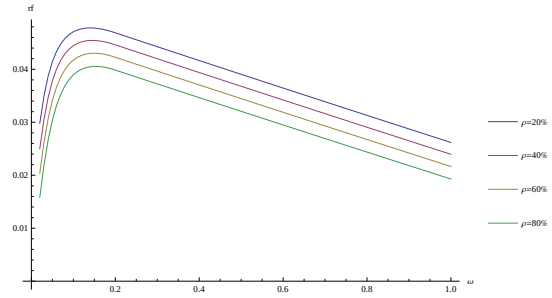
In all graphs of figure 3, the risk-free rate falls when entering the constrained region. When intermediation risk materializes, households delegation demand cannot be supported by specialist's equity contributions. For the lack of alternatives, households invest heavily into risk-free bonds and $\theta_{H,B} > 0$ (defined as an inverse function in the Appendix). This elevated savings demand pushes down the risk-free rate because relatively poor specialists are themselves reluctant to borrow. The second row of equation 2.11 illustrates the effect analytically for the "segmentation" economy. The expectation is no longer over aggregate future dividends but over terminal specialist consumption which equals their future wealth after debt repayment to households - i.e. it is levered and hence more volatile. The lower the current consumption share of specialists ω, the higher $\theta_{H,B}$ and the more dispersed the specialists future consumption. Since specialist consumption growth prices all assets, levered future consumption means volatile future marginal utility and a low risk-free rate today in return. The impact of intermedation risk is thus via specialist leverage which resonates with Adrian et al. [2011]. The effect is modulated by risk aversion and the strongest fall in the risk-free rate occurs for higher levels of risk aversion γ.

In contrast, the drop in the risk-free rate is more muted in the "partial access" economy. This is intuitive since households can maintain optimal exposure to one risky asset in the constrained region which improves risk sharing. As a consequence, households do not invest as heavily in the risk-free bond which leads to

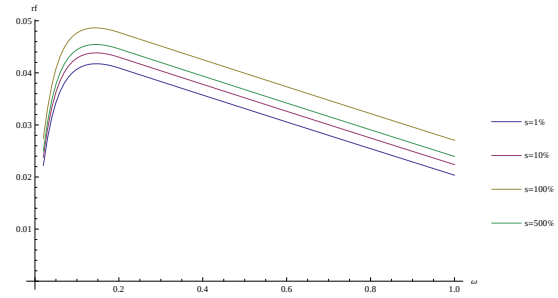
Panel A: "Segmentation" Economy



Risk Aversion γ
($s = 5$, $\rho = 40\%$)

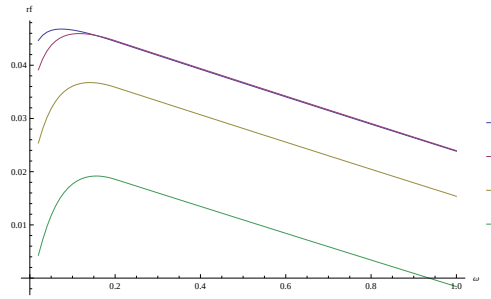


Dividend Correlation ρ
($s = 5$, $\gamma = 2$)

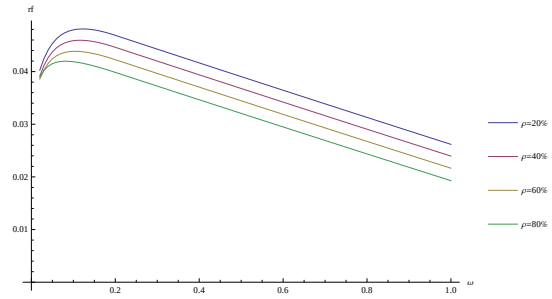


Scaling Parameter s
($\gamma = 2$, $\rho = 40\%$)

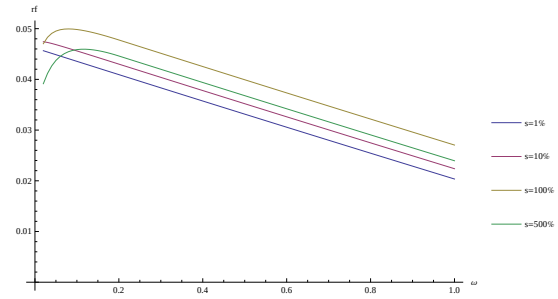
Panel B: "Partial Access" Economy



Risk Aversion γ
($s = 5$, $\rho = 40\%$)



Dividend Correlation ρ
($s = 5$, $\gamma = 2$)



Scaling Parameter s
($\gamma = 2$, $\rho = 40\%$)

FIGURE 3. **The Risk-free Rate**

This figure displays the risk-free rate in the static "segmentation" economy (panel A) and the static "partial access" economy (panel B) as a function of the endogenous state variable ω. The first graph in both panels varies risk aversion γ, the second varies the fundamental correlation between the two trees ρ and the third varies the relative sizes of the two trees via a scaling parameter s that scales the second tree relative to the first. The remaining parameters are as displayed in figure 2, the analytical expression for the risk-free rate is given in equation 2.11.

a lower increase in specialist leverage and relatively less volatile future marginal utility. The third row of equation 2.11 illustrates this insight. It is identical to the second row with the exception that the additional portfolio choice variable $\alpha_1 < 1$ works against the leverage effect in the future consumption of specialists by directly lowering the first summand inside the expectation and indirectly by being associated with lower values of $\theta_{H,B}$ in the second summand. As such, the risk-free rate is most fragile in the "segmentation" economy and more resilient in the "partial access" economy.

Fragility and resilience is even more apparent when considering risk premia. Equation 2.12 states them explicitly:

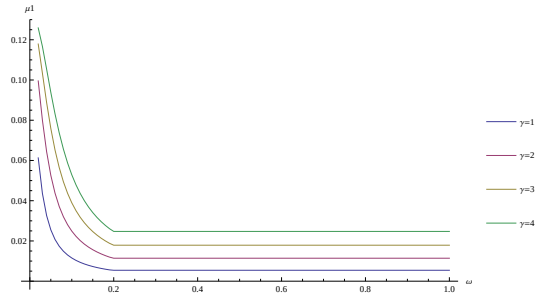
$$\mu_n(\omega) = \begin{cases} -\frac{COV\left(\delta_{n,\eta},\left(\sum_{n\in N}\delta_{n,\eta}\right)^{-\gamma}\right)}{\mathbb{E}\left[\left(\sum_{n\in N}\delta_{n,\eta}\right)^{-\gamma}\delta_{n,\eta}\right]} & if\,\omega \geq \hat{\omega} \\ -\frac{COV\left(\delta_{n,\eta},\left(\beta\left(\sum_{n\in N}\delta_{n,\eta}\right)-\theta_{H,B}\right)^{-\gamma}\right)}{\mathbb{E}\left[\left(\beta\left(\sum_{n\in N}\delta_{n,\eta}\right)-\theta_{H,B}\right)^{-\gamma}\delta_{n,\eta}\right]} & if\,\omega < \hat{\omega}\,and\,\hat{N} = \emptyset \\ -\frac{COV\left(\delta_{n,\eta},\left(\beta\left(\alpha_1\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)-\theta_{H,B}\right)^{-\gamma}\right)}{\mathbb{E}\left[\left(\beta\left(\alpha_1\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)-\theta_{H,B}\right)^{-\gamma}\delta_{n,\eta}\right]} & if\,\omega < \hat{\omega}\,and\,\hat{N} = \{1\} \end{cases} \tag{2.12}$$

where I have again dropped time subscripts. The analytical expression is insightful. When unconstrained, risk premia are independent of $\hat{N}$ and determined by the covariance between individual dividends and aggregate dividends (read aggregate consumption). The comparative statics are standard when the economy is unconstrained. For example, risk premia are increasing in risk aversion γ.

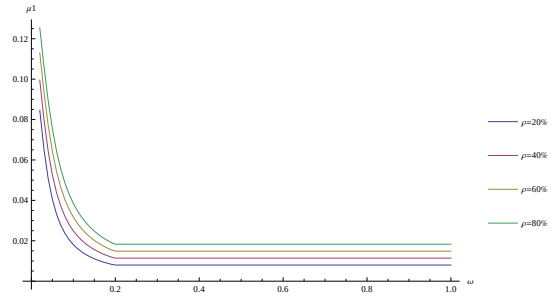
When constrained, risk premia are determined by the covariance between individual dividends and future specialist consumption which is now levered as previously described. The impact of this leverage effect strongly depends on the level of direct market access by households for the same reasons as above - direct market access allows for better risk sharing which leads to less specialist leverage during crises. In figure 4, I plot this difference for the same risky asset first when households have no access (panel A) and second when it can be accessed (panel B). The difference is stark - without direct access, its risk premium rises sharply in the constrained region. The intuition is that, by market clearing, poor and levered specialists need to hold a disproportionate amount of risk. Since portfolios cannot move, prices will and risk premia increase in order to induce specialists to hold this disproportionate amount of risk. The strong discount necessary to clear the market is a "fire sale" effect. Assets subject to "fire sales" are thus fragile.

In contrast, when the asset can be accessed by households, its risk premium is little affected by the binding capital constraint. In fact, its risk premium can even marginally decrease during crises. As such, there is little adverse effect when the fortunes of specialists deteriorate because households simply replace them in the investor base. This makes the price resilient. The marginal fall in the risk premium is intuitive once the total menu of risky assets is considered. In both configurations, there still exists an identical second risky asset to which households have no direct access.

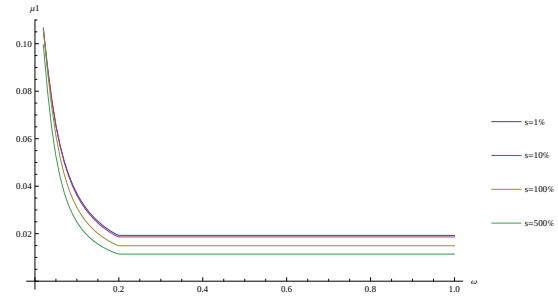
Panel A: "Segmentation" Economy



Risk Aversion γ
($s = 5$, $\rho = 40\%$)

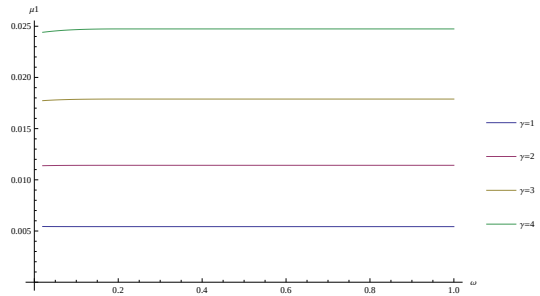


Dividend Correlation ρ
($s = 5$, $\gamma = 2$)

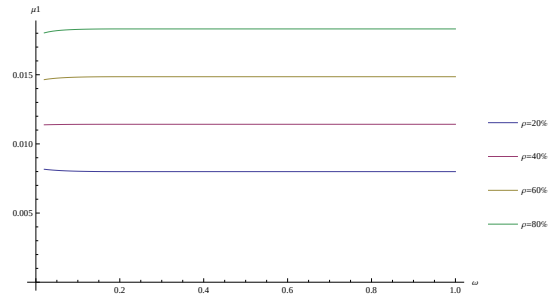


Scaling Parameter s
($\gamma = 2$, $\rho = 40\%$)

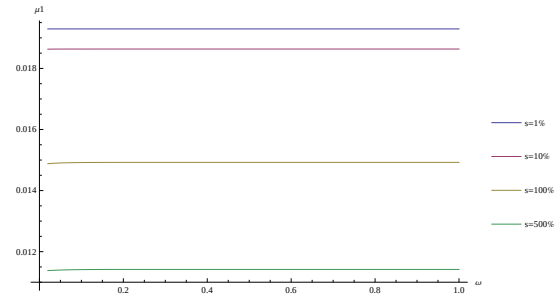
Panel B: "Partial Access" Economy



Risk Aversion γ
($s = 5$, $\rho = 40\%$)



Dividend Correlation ρ
($s = 5$, $\gamma = 2$)



Scaling Parameter s
($\gamma = 2$, $\rho = 40\%$)

FIGURE 4. **Fragility & Resilience of Risk Premia with a Variable Investor Base**
This figure displays risk premia in the static "segmentation" economy (panel A) and the static "partial access" economy (panel B) for two identical assets as a function of the endogenous state variable ω. In the "segmentation" economy, the asset can only be held by specialists, in the "partial access" economy, it can be held by both households and specialists. The first graph in both panels varies risk aversion γ, the second varies the fundamental correlation between the two trees ρ and the third varies the relative sizes of the two trees via a scaling parameter s that scales the second tree relative to the first. The remaining parameters are as displayed in figure 2, the analytical expression for risk premia is given in equation 2.12.

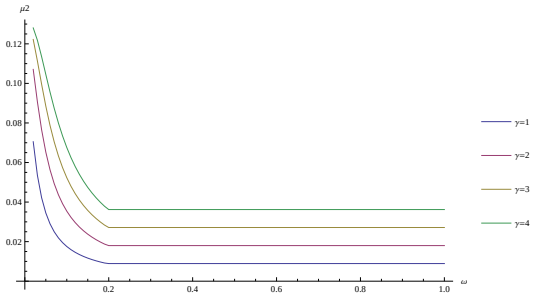
The risk premium of this asset in both cases resembles the plots in panel A of figure 4 (i.e. it experiences "fire sales"). I do not show them to conserve space. From the portfolio perspective, direct household access in one assets means that future specialist consumption is more and more tilted towards the second risky asset which needs to be held by them exclusively. This means that future cash flows of risky asset 1 are less correlated with future specialist consumption. Thus, in terms of co-movement with the pricing kernel, risky asset 1 becomes less risky as it contributes less and less to future specialist consumption. This balances off the leverage effect induced by the households bond investments and contributes to resilience.

The analysis to this point seems to suggest that, holding fundamentals constant, direct market access alone makes asset prices resilient by preventing "fire sales". Proposition 3 turns the focus on fundamental payoff characteristics holding the investor base constant.

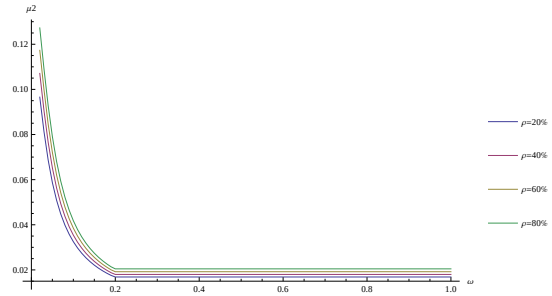
Proposition 3. *Portfolio Rebalancing Holding the investor base constant, the responses of risk premia to intermediation risk depend on portfolio rebalancing induced by fundamental risk. Asset characteristics conducive of resilience (i.e. stable or falling risk premia) include: (1) a small contribution to aggregate consumption, (2) a low fundamental correlation with other assets and (3) skewed payoffs. Assets who do not meet these characteristics tend to be fragile.*

Proposition 3 states that "fire sales" are not the sole mechanism that determines asset price fragility in the economy. In a multi-asset framework, fundamental risk interacts with intermediation risk via investors motive to rebalance. This determines which assets experience "fire sales" and which ones do not. I demonstrate the effect by plotting the risk premium of the second risky asset for the "segmentation" economy when $\hat{N} = \emptyset$ in figure 5. Panel A shows that, when the economy features two assets that are identical in every aspect, the second asset experiences "fire sales" during crises just as the first asset does in panel A of figure 4. However, the proposition identifies payoff characteristics that are conducive of falling risk premia even when both assets cannot be accessed by households. I specify the second risky asset with a different "debt-like" payoff structure. Its expected dividend growth and volatility are unchanged but the terminal payoffs are akin to debt payoffs that are identical and high in some states and decreasing in others. In other words, the payoffs are skewed differently. The three graphs in panel B of figure 5 illustrate the same comparative static exercises as in the previous case. Apart from the different payoff distributions, two asset characteristics are necessary to generate decreasing risk premia during crisis. First, the asset should be relatively small in terms of its contribution to aggregate consumption (i.e. its tree should be small). Second, it should not be excessively correlated with the second risky asset. When these conditions are met, its risk premium tends to fall and can even go negative when the economy is very constrained. In this case, only one of the two assets experiences "fire sales".

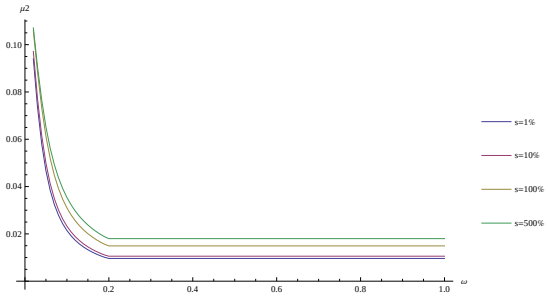
Panel A: "Segmentation" Economy - Asset 2 identical to Asset 1



Risk Aversion γ
($s = 5$, $\rho = 40\%$)

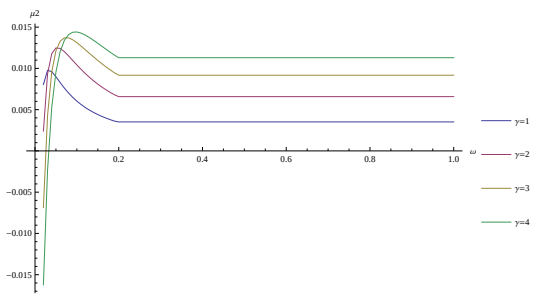


Dividend Correlation ρ
($s = 5$, $\gamma = 2$)

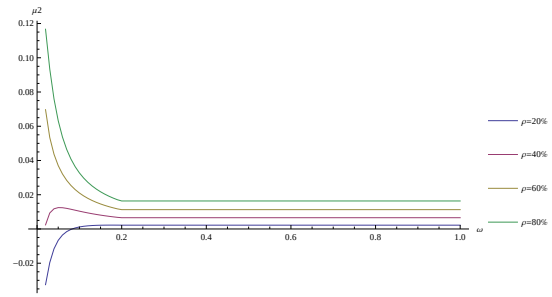


Scaling Parameter s
($\gamma = 2$, $\rho = 40\%$)

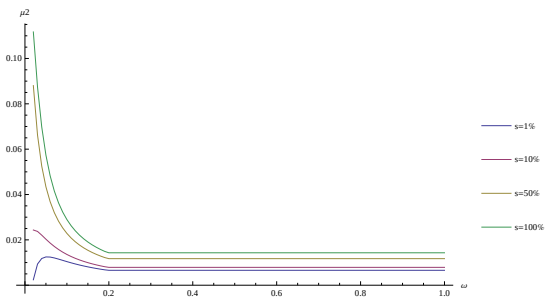
Panel B: "Segmentation" Economy - Asset 2 skewed



Risk Aversion γ
($s = 1/100$, $\rho = 40\%$)



Dividend Correlation ρ
($s = 1/100$, $\gamma = 2$)



Scaling Parameter s
($\gamma = 2$, $\rho = 40\%$)

FIGURE 5. **Fragility & Resilience of Risk Premia with a Constant Investor Base**
This figure displays risk premia in the static "segmentation" economy of the second risky assets that can only be held by specialists as a function of the endogenous state variable ω. In panel A, its payoff structure is identical to the first asset displayed in figure 4, panel A. In panel B, its payoff structure is skewed resembling the payoff structure of risky debt. The first graph in both panels varies risk aversion γ, the second varies the fundamental correlation between the two trees ρ and the third varies the relative sizes of the two trees via a scaling parameter s that scales the second tree relative to the first. The remaining parameters are as displayed in figure 2, the analytical expression for risk premia is given in equation 2.12.

The intuition behind this effect is straight-forward, a substitution effect takes place. When an asset is small with skewed payoffs that are uncorrelated with aggregate consumption, it is not very risky to begin with and displays valuable insurance features. These features are in particular high demand from specialists that are levered as holding a long position in such an asset hedges their future consumption risk and "un-levers" their net financial position. Specialists would like to overweight the asset in the intermediary which, by market clearing, is not possible. This can induce a fall in its risk premium until specialists are willing to only hold the market clearing amount. This substitution effect can make assets resilient despite their concentrated investor base.

The final price to study is the intermediation transfer F_{T-1} that serves to ration household demand when intermediation risk materializes. It is displayed in figure 6, its analytical expression is given in the Appendix. It increases strongly the more the economy is constrained. The more constrained the economy, the richer households relative to specialists. As such, households marginal utility of saving is high. Without saving opportunities, the market price of intermediation needs to rise strongly to induce suboptimal higher consumption and savings in the bond markets. In Panel B, due to direct market access in one asset, less rationing needs to take place. As such, the fee increases less dramatically compared to the case of no direct access which is in line with proposition 2.

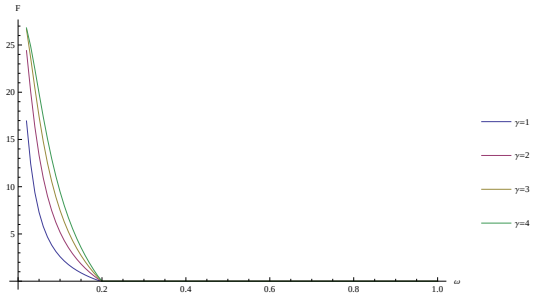
3. The Dynamic Case

In this section, I analyze equilibria in the dynamic counterparts to the static configurations studied in the previous section. The discussion there has shown that the presence of the moral hazard friction introduces state dependent transfers from households to specialists as a function of portfolio choice. This effectively means that agents face differential market prices of risk in some states of the world - a manifestation of the underlying incomplete. Standard complete market techniques are difficult to employ because consumption and portfolio choice can no longer be separated. Instead, one needs to solve for optimal consumption and portfolios jointly. To identify equilibria, I employ the recursive technique of Dumas and Lyasoff [2012] that I briefly describe before discussing the dynamic models.

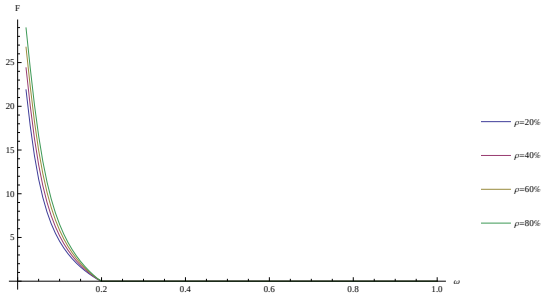
3.1. Solution Method.

The method of Dumas and Lyasoff [2012] solves incomplete financial market equilibria recursively where exogenous sources of uncertainty are represented by a multi-nomial event tree. One key difficulty their algorithm overcomes is the need to simultaneously solve for current and future consumption that shows up in the expectational equations of 2.6 and 2.7. They achieve this via two innovations.

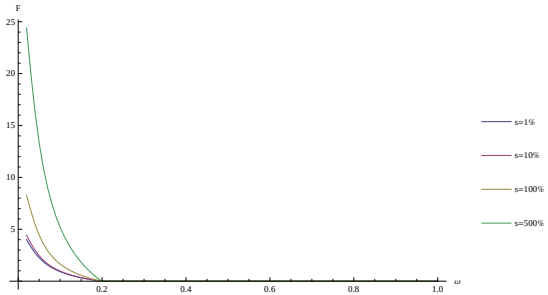
Panel A: "Segmentation" Economy



Risk Aversion γ
($s = 5$, $\rho = 40\%$)

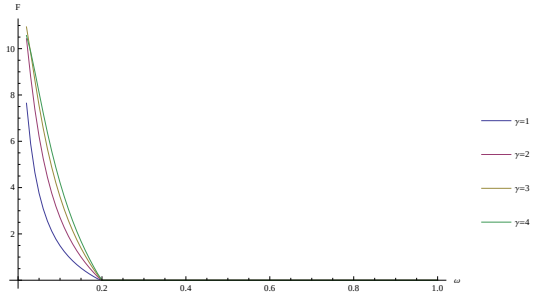


Dividend Correlation ρ
($s = 5$, $\gamma = 2$)

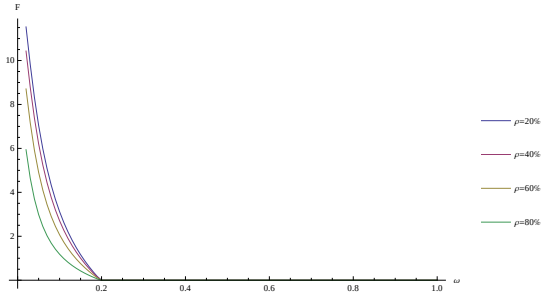


Scaling Parameter s
($\gamma = 2$, $\rho = 40\%$)

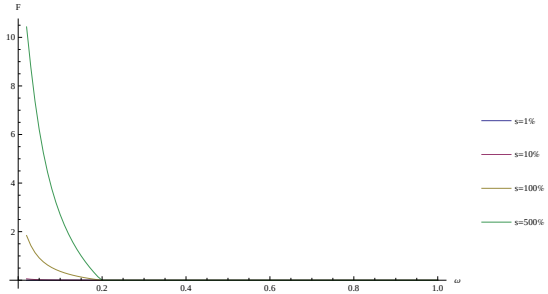
Panel B: "Partial Access" Economy



Risk Aversion γ
($s = 5$, $\rho = 40\%$)



Dividend Correlation ρ
($s = 5$, $\gamma = 2$)



Scaling Parameter s
($\gamma = 2$, $\rho = 40\%$)

FIGURE 6. **Intermediation Fee**

This figure displays the intermediation fee in the static "segmentation" economy (panel A) and the "partial access" economy (panel B) as a function of the endogenous state variable ω. The first graph in both panels varies risk aversion γ, the second varies the fundamental correlation between the two trees ρ and the third varies the relative sizes of the two trees via a scaling parameter s that scales the second tree relative to the first. The remaining parameters are as displayed in figure 2, the analytical expression for the intermediation fee is given in the Appendix.

First, they choose state prices (current consumption effectively) as state variables. Second, they shift the budget constraints of all agents one period ahead. Choosing current consumption as a state variable has the desirable feature that its range is well defined via goods market clearing. In contrast, wealth or portfolios have a priori no well defined range. Shifting the budget constraints then obviates the need to solve simultaneously for current and future consumption that show up in the Euler equations for prices. The algorithm is as follows:

(1) Specify the exogenous sources of the economy.
(2) Collect all individual optimality and market clearing conditions as well as the future budget and equilibrium constraints. Start at period $T-1$ with the terminal condition that all prices are zero.
(3) Solve for one period ahead consumption and current portfolios for all values of current consumption using an arbitrarily fine grid.
(4) Use the Euler equations to compute current prices for all the solutions of current and future consumption computed in 3.
(5) Interpolate the prices computed in 4. to obtain the current price function.
(6) Move to period $T-2$ and repeat steps 3.-5. using the known interpolated future price functions when solving for one period ahead consumption and current portfolios. Repeat until time zero is reached.
(7) At time zero, map initial endowments to current consumption.

This algorithm provides a full solution over all possible paths the economy might take. From the description of the algorithm, it is evident that the static solutions of the previous section amount to the first recursion of the algorithm. The first recursion can be solved analytically because future prices (including fees) need not be taken into account (they collapse to zero due to the terminal condition). This provided the benefit of studying some asset pricing quantities (i.e. risk premia, the risk-free rate) analytically at the cost of sacrificing others that require a dynamic context (volatility, correlation). The dynamic solutions presented now fill this gap.

To illustrate, consider the "segmentation" economy with N risky assets and a bond and one population each of households and specialists. Following He [1990], the exogenous sources of uncertainty (N dividends) can be represented by a $(N+1)$-nomial tree where every node in the tree has $(N+1)$ successor nodes, that I index by η. The fundamental cash flow processes of equation 2.1 are calibrated on this partially recombining tree. In a general equilibrium model, this is a very convenient result as it does not change the completeness

Equations		Unknowns	
Future Budget Constraints	$2 \times (N+1)$	Future Consumption	$2 \times (N+1)$
Market Clearing	$N+2$	Portfolio Choices	$(N+2)+2$
Price Agreement	2		
Euler Equations for Prices	$N+1$	Prices	$N+1$
Capital Constraint	1	Fee	1
Total	$4N+8$	Total	$4N+8$

FIGURE 7. **Time t Equation System for the "Segmentation" Economy with N Assets**

of the market. The equations that characterize equilibria at every point in time are already familiar at this stage. They include

- The first order conditions of equations 2.6 and 2.7 written at time t where future prices are not dropped
- Market clearing in all security markets, the market for intermediation as well as the goods market
- The capital constraint of equation 2.4
- The budget constraints of all agents

The algorithm is operationalized by shifting the budget constraints one period ahead and recognizing that households and specialists need to agree on prices for securities to which they enjoy common access as discussed in section 2.1 (the risk-free bond and the intermediary portfolio in the current example). The system of equations and unknowns is summarized in figure 7 for the "segmentation" economy with N risky assets. This system is solved recursively. In practical terms, the actual number of equations that need to be solved for every grid point can be reduced by substituting out market clearing and the budget constraints in the price agreement conditions and computing current prices after future consumption is known leaving a much smaller system that can be conveniently handled.[3]

3.2. **Contagion and Decoupling.** The main insight in this section is that asset price fragility (resilience) from the static analysis translates into contagion (decoupling) in the dynamic analysis. I directly state the results in two propositions.

Proposition 4. *Fragility and Contagion During crises, fragile assets tend to exhibit contagion whereas resilient assets tend to decouple.*

Proposition 5. *Portfolio Rebalancing and Contagion Payoff characteristics of asset pairs conducive of fragility (resilience) as identified in proposition 3 are associated with contagion (decoupling) during crises.*

[3]One complication that arises is the inequality constraint. The interior point algorithm of Armand et al. [2006] offers a convenient solution to incorporate inequality constraints into the algorithm.

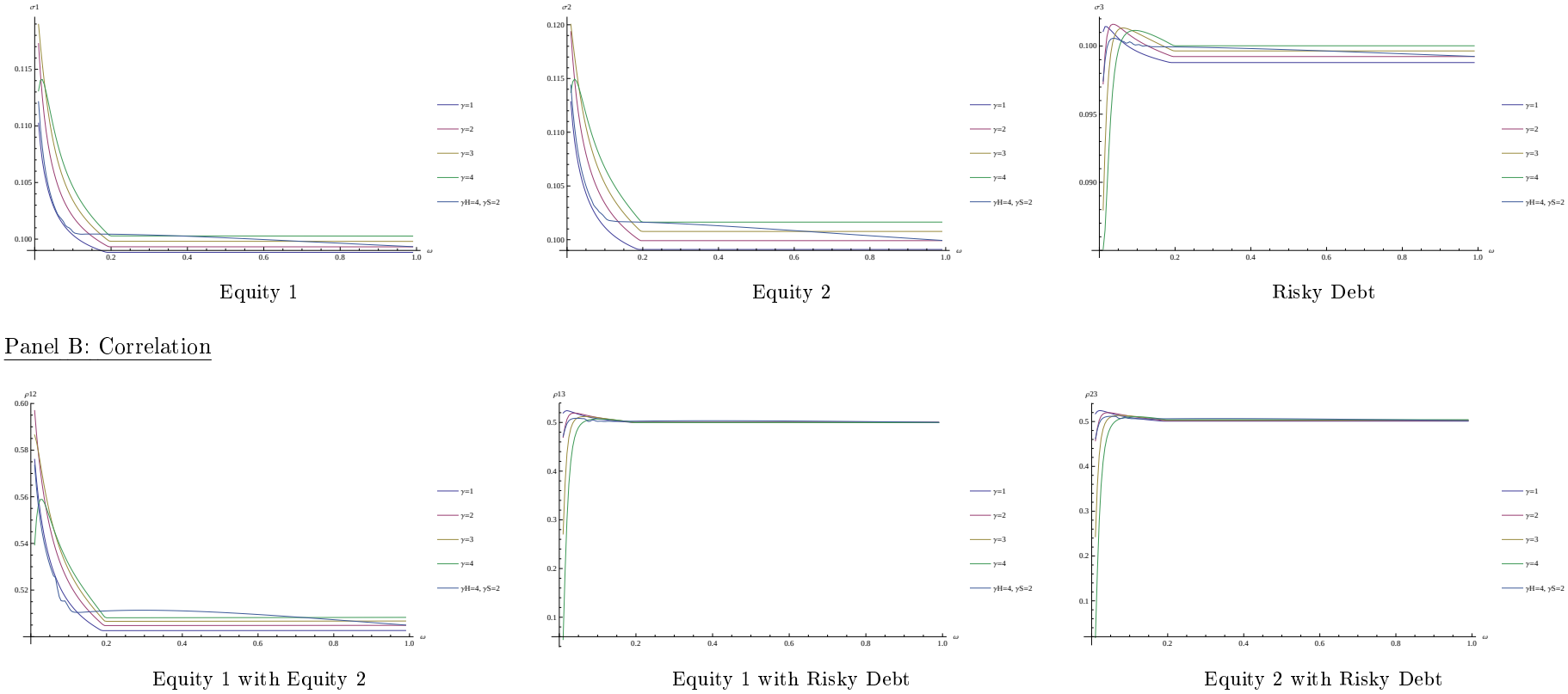
To demonstrate the main insights, I immediately introduce a richer asset menu. Specifically, the set of securities comprises a short term risk-free bond, and three risky securities. Two of them are identical in every aspect, I label them "equity" for easier reference. The third has a payoff distribution that is skewed, I label it "risky debt". In the "segmentation" economy, as before, households only access the risk-free bond and delegate funds to specialists; in the "partial access" economy, they may additionally access one of the identical equities directly. The parameter specifications are as in the previous examples. The results displayed are the time zero solutions in economies with a time horizon of 5 periods. As a baseline, I set the fundamental dividend correlation between every individual tree at 50%, the initial tree sizes have a ratio of $Equity\,1 - Equity\,2 - Risky\,Debt = \frac{1}{2} - 1 - \frac{1}{100}$ in order to induce the spillover and substitution effects demonstrated before. That is, the risky debt security has not only skewed payoffs but a small contribution to aggregate consumption which was shown to be important for resilient risk premia with a constant investor base. The main parameter I vary is risk aversion γ where I now also allow $\gamma_S \neq \gamma_H$ and in particular $\gamma_S < \gamma_H$.

The algorithm reproduces the analytical results from the previous section on the risk-free rate and the risk premia. I do not show them here to conserve space, they are available upon request. Instead, I directly focus on volatility and co-movement. Since fundamentals are i.i.d., any crises induced change in the dynamic behavior of asset prices can be attributed to contagion or decoupling, i.e. these changes are in excess of their fundamental levels. Hence, I define positive (negative) excess correlation as contagion (decoupling).

I begin by analyzing contagion and decoupling in the "segmentation" economy without direct household access. The static analysis identifies the two identical equities as being fragile because they tend to experience "fire sales" at the same time. In contrast, risky debt was identified as resilient because of its insurance properties. Portfolio rebalancing towards that asset made its price process resilient during crises. Propositions 4 and 5 therefore predict contagion between the two equities and decoupling of risky debt. Figure 8 displays that this is indeed the case. Volatility of the two equities and the correlation between them increase in the constrained region between 10-20% relative to their fundamental levels. Risky debt, however, decouples - the volatility of its risk premium falls by up to 10%, its correlation with the other two equities drops between 10% and over 90% depending on the level of risk aversion.

In the "partial access" economy, one of the two equities is now resilient due to direct household access. Proposition 4 predicts that this asset decouples from its identical counterpart that does not share the same flexibility in its investor base. Proposition 5 still predicts decoupling between the second equity and risky debt because they share a common investor base. Hence, introducing direct household access makes contagion less likely. Figure 9 shows that this is exactly the case. From the static analysis, we know that in the constrained region, the first equity is increasingly held by households and its risk premium hardly moves. It falls

Panel A: Volatility



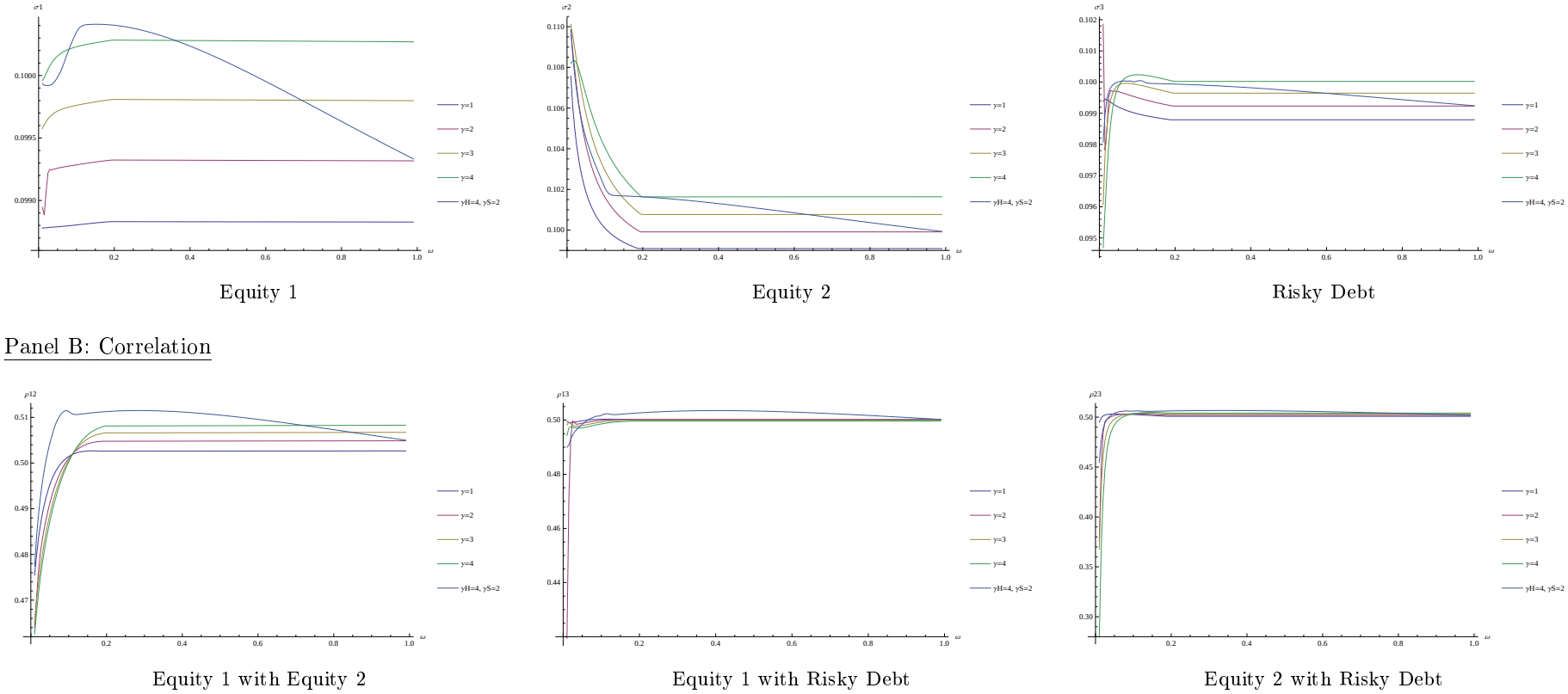
Equity 1 | Equity 2 | Risky Debt

Panel B: Correlation

Equity 1 with Equity 2 | Equity 1 with Risky Debt | Equity 2 with Risky Debt

FIGURE 8. **Contagion and Decoupling when $\hat{N} = \emptyset$**
This figure displays time zero volatilities (panel A) and correlations (panel B) of risk premia in the "segmentation" economy as a function of the endogenous state variable ω. Equities 1 and 2 are identical in every aspect except for their initial tree sizes. Risky debt has skewed payoffs as described in the text. None of those securities can be directly accessed by households. The economy has a horizon of 5 periods, the fundamental dividend correlation between every individual tree is set to 50% and the initial tree sizes have a ratio of $\frac{1}{2} - 1 - \frac{1}{100}$. The remaining parameters are as displayed in figure 2.

Panel A: Volatility



Equity 1 | Equity 2 | Risky Debt

Panel B: Correlation

Equity 1 with Equity 2 | Equity 1 with Risky Debt | Equity 2 with Risky Debt

FIGURE 9. **Contagion and Decoupling when $\hat{N} = \{1\}$**
This figure displays time zero volatilities (panel A) and correlations (panel B) of risk premia in the "partial access" economy as a function of the endogenous state variable ω. Equities 1 and 2 are identical in every aspect except for their initial tree sizes. Risky debt has skewed payoffs as described in the text. Only Equity 1 can be directly accessed by households. The economy has a horizon of 5 periods, the fundamental dividend correlation between every individual tree is set to 50% and the initial tree sizes have a ratio of $\frac{1}{2} - 1 - \frac{1}{100}$. The remaining parameters are as displayed in figure 2.

marginally as specialists have a motive to diversify their portfolio that is increasingly concentrated in the remaining assets. As such, the volatility of risky asset 1 falls marginally. Risky asset 2 experiences heavy "fire sales" and its volatility increases. The substitution effect inside the portfolio leads to lower volatility of risky debt. This means that all assets decouple from each other, albeit for different reasons. The correlation of the first equity and both other asset falls because the investor base in the former is replaced while the base in the latter is held constant. Since one asset price stays largely constant and the other two move, the assets decouple. Inside the intermediary portfolio, the large equity decouples from the small risky debt because of the substitution motive of specialists.

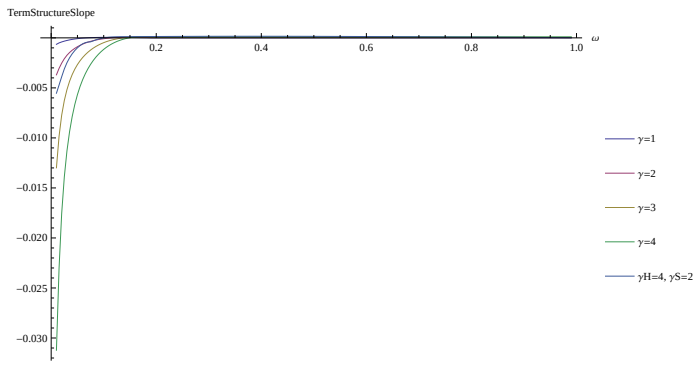
The graphs presented in this sub-section include one example with differential risk aversion between households and specialists. I have assumed that households are more risk averse than specialists. Introducing differential risk aversion causes proposition 1 to break down[4] - bond markets are no longer dormant when the economies are unconstrained and the financial sector always carries some leverage as specialists borrow from more risk averse households. Further, households generally have a non-zero position in their directly-accessible securities. This change in set-up induces some changes in equilibrium quantities. For example, risk premia are no longer flat in the unconstrained region over the domain of the endogenous state variable ω. The higher the current consumption share of households, the higher risk premia since the average risk aversion in the economy increases. Another difference is that with differential risk aversion, the threshold when the economy becomes constrained moves considerably to the left in the graphs. Households demand relatively less risk exposure than do specialists for the same level of wealth. This means that the capital constraint starts binding later. In addition, the graphs tend to display some "wiggles", mostly around the threshold when the economy becomes constrained and in the constrained region. This is partly a manifestation of the fragility of the economy when approaching the lower end of the state space and partly a result of the fact that around the threshold, future states tend to be more dispersed and fall either further to the left or the right of the future thresholds. The "wiggles" often disappear once sufficiently deep in the constrained region where all future states are also constrained.

3.3. **Two Additional Experiments.** In this section, I run two additional experiments to see if the basic mechanics of the model relate to other stylized facts of financial markets.

An artificial term structure via derivatives. I begin by pricing a derivative that mimics long-term risk-less debt that has a unit payoff in all terminal states only but no intermediate payoffs. Thereby, I create an artificial term-structure. I assume that only specialists have access to the derivative. This experiment fully

[4]Strictly speaking, in the dynamic context when investors are more risk averse than a log investor, the economies are not exactly independent of direct market access by households due to intertemporal hedging of future fees. However, from an inspection of figures 8 and 9, this effect is likely small. As such, the main point remains that absent a materialization of intermediation risk, equilibria are not (or little) affected by limited direct household participation.

exploits the substitution motive in the intermediary portfolio. Since specialists are both poor and forced to lever up during crisis, the derivative should experience strong demand because it is a good hedge and "un-levers" the specialists financial positions. I present the result in figure 10 for the "segmentation" economy.

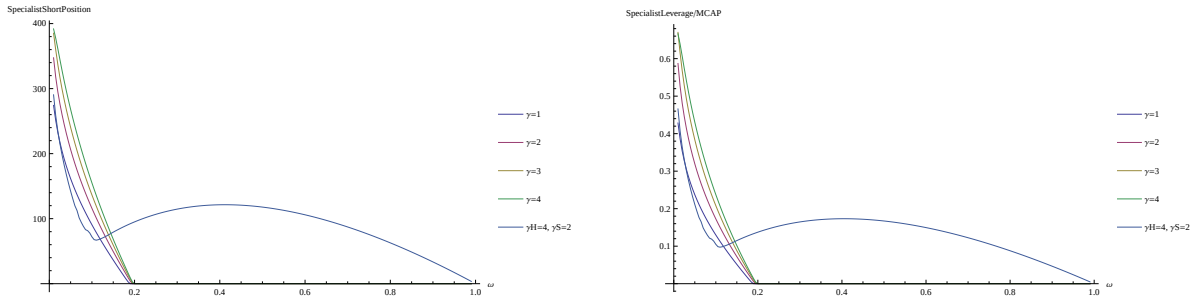


Slope of Term Structure (5 period horizon)

FIGURE 10. **An Artificial Term Structure in the "Segmentation" Economy**
The figure displays an artificial term structure generated by pricing a derivative in zero net supply that has a unit payoff in all states in the terminal period but no intermediate cash flows. The derivative is assumed to be accessible by specialists only in the "segmentation" economy. The figure graphs the slope of the term structure computed as the excess return of the derivative over the risk-free rate as a function of the endogenous state variable ω. Parameter specifications are as described in section 3.2.

The graph shows the term structure slope as the difference between the expected return on the derivative and the risk-free rate. Indeed, while this artificial term structure is flat in the unconstrained region, it inverts during crisis. This is a purely financial effect since fundamentals do not change. The effect is still present in the "partial access" economy, albeit less dramatic. The inversion of this artificial yield curve is increasing in risk aversion.

Differential risk aversion and financial sector leverage. In figure 11, I graph financial sector leverage (i.e. specialist borrowing). With equal risk aversion, bond markets are dormant in the unconstrained region as shown in proposition 1. Specialist leverage, either in terms of bond holdings or value of debt by total market capitalization, increases strongly in the constrained region for reasons familiar by now. Introducing differential risk aversion now leads bond markets to become active at all times. I consider the case when specialists are less risk averse than households ($\gamma_S = 2$, $\gamma_H = 4$). In this case, the financial sector is levered at all times. Initially, when specialists are dominant in the economy, leverage is increasing in household consumption share. In other words, during good time the financial sector levers up. When approaching the capital constraint and households become more dominant, specialists reduce leverage. Once the capital constraint binds and households are shut out from risky asset markets, leverage increases strongly. As such, differential risk aversion appears helpful in generating a basic leverage pattern for the financial sector.



Specialist borrowing (bond holdings) Specialist Leverage by Market Capitalization

FIGURE 11. **Financial Sector Leverage in the "Segmentation" Economy**
The figure displays financial sector leverage both in terms of specialists bond holdings and market value of leverage by total market capitalization for the "segmentation" economy as a function of the endogenous state variable ω. Parameter specifications are as described in section 3.2.

4. Discussion and Extensions

In this section, I discuss how the main model presented in this paper can be extended. First, I detail how the set-up is sufficiently general to accommodate more than two populations of investors that contract with each other. Second, I propose an extension that promises more realism in the analysis.

4.1. **The Case of K Populations.** I line out how the framework I have presented can be generalized to the case with K populations that allows for general contracting between different classes of agents. Such a general framework promises interesting along several dimensions.

(1) It allows for multiple specialist populations that are allowed to contract with each other on possibly differentiated products.
(2) The multiple investor populations give rise to differentiated prices of intermediation services depending on who chooses to contract with whom in equilibrium.
(3) These two features allow a more detailed modeling of financial intermediation and risk sharing by, e.g. focusing only on the relationships between different kinds of specialists as opposed to the relationships between specialists and unsophisticated households.

Consider an economy populated by K populations of agents with N risky assets with time discrete from $t = 0, ..., T$. A representative agent i_k from some population $k \in K$ can access a subset $N^k \subset N$ of risky assets but can simultaneously contract with as many agents from other populations k'.[5] Under the same

[5]One can think of several reasons why agent k only accesses a subset of risky assets. For instance, imagine obtaining direct access entails a fixed costs of setting up trading desks and some subsequent maintenance costs as in Brennan [1975] or Allen and Gale [1994]. The benefits of direct access are twofold. First, the agent can sell the access to other agents to generate fee income. Second, he can trade in the asset himself in order to insure his consumption stream. The choice of establishing and

type of moral hazard, a set of capital constraints as in equation 2.4 will obtain. Agent i_k will demand from his contracting partners to retain enough "skin in the game" for the products he purchases. Likewise, agents $i_{k'}$ will demand from agent i_k to contribute enough equity to be properly incentivized to work himself. The Lagrangian for the population k is stated in Appendix E. The program E.1 is the generalized form of the programs underlying the previous sections. Agents maximize with respect to their own consumption, their contribution to the intermediary they manage, the portfolio composition of that intermediary, how much to delegate to other agents as well as risk-free savings. For the savings they delegate to other agents, they set incentives by imposing the capital constraint on others. Notice that this leads to differentiated fees, both on the paying and the receiving end of agents. That is, the transfers charged between every pair of populations will generally differ. To see why, consider the associated optimality conditions that characterize equilibrium where the notation is as before and I^k refer to the intermediaries offered by population k.

$$
\begin{aligned}
U_k'\left(c_{k,t}\right) &= \phi_{k,t} \\
\mathbb{E}_t\left[\phi_{k,t+1}\left(\sum_{n^k=1}^{N^k}\alpha_{n^k,t}\left(P_{n^k,t+1}+\delta_{n^k,t+1}\right)\right)\right] &= \phi_{k,t}\left(\sum_{n^k=1}^{N^k}\alpha_{n^k,t}P_{n^k,t}\right) \\
\mathbb{E}_t\left[\phi_{k,t+1}\left(P_{n^k,t+1}+\delta_{n^k,t+1}\right)\right] &= \phi_{k,t}P_{n^k,t}\,\forall n^k\in N^k \\
\mathbb{E}_t\left[\phi_{k,t+1}\left(\sum_{n^{k'}=1}^{N^{k'}}\alpha_{n^{k'},t}\left(P_{n^{k'},t+1}+\delta_{n^{k'},t+1}\right)\right)\right] &= \phi_{k,t}\left(\sum_{n^{k'}=1}^{N^{k'}}\alpha_{n^{k'},t}P_{n^{k'},t}+F_{k,I^{k'},t}\right)+\phi^{CC}_{k,k',t} \\
\mathbb{E}_t\left[\phi_{k,t+1}\right] &= \phi_{k,t}B_t \\
\phi^{CC}_{k,k',t}\left(\beta-\theta_{k',I^{k'},t}\right) &= 0\,\forall I^{k'} \\
\theta_{k',I^{k'},t} &\geq \beta\,\forall I^{k'} \\
& B.C.
\end{aligned}
\tag{4.1}
$$

Differentiated fees arise because price agreement needs to hold between the agents who purchase exposure and the agents who sell it. The condition when population k purchases from population k' is

$$
\mathbb{E}_t\left[\frac{\phi_{k,t+1}}{\phi_{k,t}}\left(\sum_{n^{k'}=1}^{N^{k'}}\alpha_{n^{k'},t}\left(P_{n^{k'},t+1}+\delta_{n^{k'},t+1}\right)\right)\right]-\frac{\phi^{CC}_{k,k',t}}{\phi_{k,t}}=\mathbb{E}_t\left[\frac{\phi_{k',t+1}}{\phi_{k',t}}\left(\sum_{n^{k'}=1}^{N^{k'}}\alpha_{n^{k'},t}\left(P_{n^{k'},t+1}+\delta_{n^{k'},t+1}\right)\right)\right]
\tag{4.2}
$$

hence the fee population k will have to pay to population k' will depend on the attractiveness of the product k' offer to k. The impediment that does not allow for equalization of fees across agent populations is

maintaining access will be a trade-off between such costs and benefits. Incorporating such decisions may be interesting in itself yet appears a great challenge.

the limitation that agents across populations cannot form coalitions when they purchase exposure from population k'.

What could be achieved under such a generalized framework? Assuming the presence of different financial institutions that intermediate across assets (e.g. pension funds that directly invest in equity and corporate debt, say, but hire alternative asset managers for other assets. Alternative asset managers themselves delegate to pension funds), one could study risk sharing among financial institutions rather than between financial institutions and households which appears more direct in light of the current crisis. One criticism of the results of the previous sections is that crises occur when the financial sector (specialists) are small relative to households. The current crisis unfolded when the financial sector was very big relative to the rest of the economy which appears challenging. Such a challenge could be reconciled by focusing on the connectedness of different financial institutions, where crises do not depend on the size of the financial sector but the concentration of risk within it. Interesting aspects of such a structure include the concentration of leverage across financial firms as well as portfolio overlaps that result from contracting between financial firms and associated asset pricing implications.

4.2. **Continuous Effort and Optimal Shirking.** One extension could refine the binary "work or shirk" decision into a continuous effort choice. This appears desirable because the current model requires strong parameter restrictions such that households always wish to implement "working" (see the discussion in Appendix B). It seems realistic that at some point, it is less expensive to let the specialist "shirk" instead of paying him to "work". However, such an analysis would require a richer modeling of moral hazard with potentially other costly mechanisms that specialists can use to signal their good intentions. For example, one avenue could introduce the possibility that specialists invest in "internal audits" that are costly but alleviate moral hazard. The analysis could then focus on optimal shirking, corporate governance and asset pricing as a function of endogenous states.

5. Concluding Remarks

I analyze the interplay between fundamental and intermediation risk in a dynamic multi-asset economy in which heterogeneous investors overcome their limited market access by contracting with each other. This contracting is impacted by a moral hazard friction. The model shows that dynamic asset pricing behavior depends both on fundamental and intermediation risk as well as the interaction between the two. The central point of the paper is that these interactions gain significance in certain states of the world. For possibly large regions in the state space, the incompleteness associated with limited market access and intermedation frictions has no or little equilibrium impact. As such, a complete understanding of the dynamic behavior

of asset prices requires a complete understanding of both fundamental cash flow risk and non-fundamental intermediation risk that is associated with the institutional characteristics of markets and the frictions that follow from those characteristics.

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Appendix A. Static Problem Formulation

The specialists Lagrangian at $T-1$ is

$$\mathbb{L}_{S,T-1} = \max_{\left\{c_{S,T-1},\theta_{S,I,T-1},\left\{\alpha_{n,T-1}\right\},\theta_{S,B,T-1}\right\}} U_S\left(c_{S,T-1}\right) + \mathbb{E}_{T-1}\left[U_S\left(c_{S,T}\right)\right] \tag{A.1}$$

$$\begin{aligned}
&-\phi_{S,T-1}\left(c_{S,T-1} + \theta_{S,I,T-1}\left(\sum_{n=1}^{N}\alpha_{n,T-1}P_{n,T-1}\right) + \theta_{S,B,T-1}B_{T-1} - \left(1-\theta_{S,I,T-1}\right)F_{T-1}\right)\\
&+\phi_{S,T-1}\left(\theta_{S,I,T-2}\left(\sum_{n=1}^{N}\alpha_{n,T-2}\left(P_{n,T-1}+\delta_{n,T-1}\right)\right) + \theta_{S,B,T-2}\right)\\
&-\mathbb{E}_{T-1}\left[\phi_{S,T}\left(c_{S,T} - \left(\theta_{S,I,T-1}\left(\sum_{n=1}^{N}\alpha_{n,T-1}\delta_{n,T}\right) + \theta_{S,B,T-1}\right)\right)\right]
\end{aligned}$$

The households Lagrangian equivalently at $T-1$ is

$$\begin{aligned}
\mathbb{L}_{H,T-1} = &\max_{\left\{c_{H,T-1},\theta_{S,I,T-1},\theta_{H,B,T-1},\hat{\alpha}_{n,T-1}\right\}} U_H\left(c_{H,T-1}\right) + \mathbb{E}_{T-1}\left[U_H\left(c_{H,T}\right)\right]\\
&\phi_{H,T-1}\left(c_{H,T-1} + \left(1-\theta_{S,I,T-1}\right)\left(\sum_{n=1}^{N}\alpha_{n,T-1}P_{n,T-1} + F_{T-1}\right) + \sum_{n=1}^{\hat{N}}\hat{\alpha}_{n,T-1}P_{n,T-1} + \theta_{H,B,T-1}B_{T-1}\right)\\
&+\phi_{H,T-1}\left(\left(1-\theta_{S,I,T-2}\right)\left(\sum_{n=1}^{N}\alpha_{n,T-2}\left(P_{n,T-1}+\delta_{n,T-1}\right)\right) + \sum_{n=1}^{\hat{N}}\hat{\alpha}_{n,T-2}\left(P_{n,T-1}+\delta_{n,T-1}\right) + \theta_{H,B,T-2}\right)\\
&-\mathbb{E}_{T-1}\left[\phi_{H,T}\left(c_{H,T} - \left(\left(1-\theta_{S,I,T-1}\right)\left(\sum_{n=1}^{N}\alpha_{n,T-1}\delta_{n,T}\right) + \sum_{n=1}^{\hat{N}}\hat{\alpha}_{n,T-1}\delta_{n,T} + \theta_{H,B,T-1}\right)\right)\right]\\
&-\phi_{T-1}^{CC}\left(\beta - \theta_{S,I,T-1}\right)\\
&\theta_{S,I,t} \geq \beta
\end{aligned} \tag{A.2}$$

Appendix B. Intermediation Equilibrium

The symmetry of the contracting equilibrium is as in He and Krishnamurthy [2012b]. Intuitively, if the equilibrium were asymmetric with some households obtaining different risk exposures and some specialists earning different fees, one could assemble a coalition of households and specialists who are worse off compared to other household-specialist pairs and improve. Such deviating coalitions are ruled out when the equilibrium is symmetric.

To implement $s_{T-1} = 0$, the capital constraint in equation 2.4 needs to hold, i.e. $\theta_{S,I,T-1} \geq \beta$ as discussed in the main body. That the transfer takes the form $F_{T-1} = \frac{\phi^{CC}_{T-1}}{\phi_{H,T-1}}$ can be seen from Equation 2.8. This equation can be interpreted as equating demand and supply for risk exposure since both come from optimality conditions on portfolio choice and hence represent demand curves. Demand from specialists therefore leaves the residual exposure as supply to households. The free term is $\frac{\phi^{CC}_t}{\phi_{H,t}}$, so $\frac{\phi^{CC}_t}{\phi_{H,t}}$ plays the role of equating intermediation demand with supply and must thus be the price of intermediation services, F_t.

To obtain that households always wish to implement "working", the shared dollar cost of "shirking" needs to be high. I derive a parameter restriction on the moral hazard primitives. Assume one individual household that considers implementing "shirking", i.e. $s_{T-1} = 1$, for instance because he cannot obtain his first-best level of risk exposure. Such a situation would occur when the capital constraint binds and household demand is rationed. I assume this household is marginal without impact on equilibrium prices or policies of others. The benefit of implementing $s_{T-1} = 1$ include a higher risk exposure today with associated expected cash flows in the terminal period. This household would wish to implement some alternative sharing rule $\theta_{S,\bar{I},T-1}$ such that his terminal risky cash flows become

$$\mathbb{E}^{\mathbb{H}}{}_{T-1}\left[\left(1-\theta_{S,\bar{I},T-1}\right)\sum_{n=1}^{N}\alpha_{N,T-1}\delta_{n,T,\eta}\right]$$

as opposed to

$$\mathbb{E}^{\mathbb{H}}{}_{T-1}\left[\left(1-\theta_{S,I,T-1}\right)\sum_{n=1}^{N}\alpha_{N,T-1}\delta_{n,T,\eta}\right]$$

The incremental benefit is thus (in expectation)

$$\Delta Benefit = \mathbb{E}^{\mathbb{H}}{}_{T-1}\left[\left(\theta_{S,I,T-1}-\theta_{S,\bar{I},T-1}\right)\sum_{n=1}^{N}\alpha_{N,T-1}\delta_{n,T,\eta}\right] \tag{B.1}$$

where the expectation is taken with respect to the pricing kernel of the household (i.e. his consumption growth).

To obtain this incremental benefit, the household would offer the alternative fee $F^{\bar{}}_{T-1}$to the specialist and accept shirking which will bring an additional cost of $\left(1-\theta_{S,\bar{I},T-1}\right)X_{T-1}$. This alternative fee needs to make the specialist indifferent between accepting the alternative sharing rule and starting a relationship with another household at equilibrium conditions. As such, the household must offer at least

$$F^{\bar{}}_{T-1} = F_{T-1} - \left(Z_{T-1} - \theta_{S,\bar{I},T-1}X_{T-1}\right)$$

whereas under the equilibrium contract, he would simply pay F_{T-1}. That is the incremental cost of implementing shirking is

$$\begin{aligned}
\Delta Cost &= F_{\bar{T-1}} + \left(1 - \theta_{S,\bar{I},T-1}\right) X_{T-1} - F_{T-1} \\
&= F_{T-1} - \left(Z_{T-1} - \theta_{S,\bar{I},T-1} X_{T-1}\right) + \left(1 - \theta_{S,\bar{I},T-1}\right) X_{T-1} - F_{T-1} \\
&= X_{T-1} - Z_{T-1} \qquad \text{(B.2)}
\end{aligned}$$

Comparing equations B.1 and B.2 shows that implementing shirking always yields a fixed costs that depends on the primitives of the moral hazard problem. In contrast, the additional benefit of letting the specialist shirk depend on aggregate states since portfolio choices depend on aggregate states (see Appendices C and D below). I assume that X_{T-1} is sufficiently large such that implementing $s_{T-1} = 0$ is always possible. The maximum benefit in the static case occurs when households wished to implement $\theta_{S,\bar{I},T-1} = 0$ instead of $\theta_{S,I,T-1} = \beta$ for the situation where households have no direct access (i.e. the segmentation economy). Hence a sufficient condition in the static case is

$$X_{T-1} \geq \mathbb{E}^{\mathbb{H}}{}_{T-1} \left[\beta \sum_{n=1}^{N} \delta_{n,T,\eta} \right] + Z_{t-1} = X_{\bar{T-1}} \qquad \text{(B.3)}$$

Evidently, in the dynamic case where the expectation also includes future prices, this sufficient condition would no longer hold, X_{T-1} would need to grow bigger. In fact, for this extreme case, the marginal benefit of letting the specialist shirk but obtaining higher risk exposure would ultimately grow towards infinity (this is also the case in He and Krishnamurthy [2012b], see equations A.8 and A.9). For the limited time horizons I analyze in section 3, I assume that this region is negligibly small and abstract from this complication. Future work would have to address how much shirking is optimal as a function of state variables and under what conditions households accept shirking as a lower cost compared to implementing working and payer a higher explicit market price.

Appendix C. Analytical Solution When $\hat{N} = \emptyset$

The equation system when $\hat{N} = \emptyset$ (the "segmentation" economy) and equal risk aversion after collecting all individual optimality conditions 2.6 and 2.7, substituting market clearing conditions and future consumption via budget constraints, using the relationship on the fee from lemma 1, dropping time subscripts (future random variables have subscript η referring to future states at time T) and writing shorthand θ_S for $\theta_{S,I}$ and θ for $\theta_{H,B}$ becomes

$$
\begin{aligned}
\mathbb{E}\left[e^{-\rho_S}\left(\frac{\omega\left(\sum_{n\in N}\delta_n\right)}{\theta_S\left(\sum_{n\in N}\delta_{n,\eta}\right)-\theta}\right)^{\gamma}\right] &= \mathbb{E}\left[e^{-\rho_H}\left(\frac{(1-\omega)\left(\sum_{n\in N}\delta_n\right)}{(1-\theta_S)\left(\sum_{n\in N}\delta_{n,\eta}\right)+\theta}\right)^{\gamma}\right] \\
\mathbb{E}\left[e^{-\rho_S}\left(\frac{\omega\left(\sum_{n\in N}\delta_n\right)}{\theta_S\left(\sum_{n\in N}\delta_{n,\eta}\right)-\theta}\right)^{\gamma}\left(\sum_{n\in N}\delta_{n,\eta}\right)\right] &= \mathbb{E}\left[e^{-\rho_H}\left(\frac{(1-\omega)\left(\sum_{n\in N}\delta_n\right)}{(1-\theta_S)\left(\sum_{n\in N}\delta_{n,\eta}\right)+\theta}\right)^{\gamma}\left(\sum_{n\in N}\delta_{n,\eta}\right)\right]-F \\
F\left(\beta-\theta_S\right) &= 0 \qquad \text{(C.1)} \\
\theta_S &\geq \beta
\end{aligned}
$$

where the endogenous state variable is ω, the current consumption share of the specialist and the unknowns are $\{\theta,\, \theta_S,\, F\}$. The solution to these equations includes two sub-cases, determined by the complementary slackness condition in the third row above. First, $\{F = 0,\, \theta_S > \beta\}$ and second, $\{F > 0,\, \theta_S = \beta\}$.

C.1. **Case** $\{F = 0,\, \theta_S > \beta\}$**.** Guess $\theta = 0$. Use the first equation of system C.1 and solve for θ_S.

$$
\begin{aligned}
\mathbb{E}\left[e^{-\rho_S}\left(\frac{\omega}{\theta_S\left(\sum_{n\in N}\delta_{n,\eta}\right)}\right)^{\gamma}\right] &= \mathbb{E}\left[e^{-\rho_H}\left(\frac{(1-\omega)}{(1-\theta_S)\left(\sum_{n\in N}\delta_{n,\eta}\right)}\right)^{\gamma}\right] \\
\theta_S &= \left[\left[\frac{\mathbb{E}\left[e^{-\rho_H}\left(\sum_{n\in N}\delta_{n,\eta}\right)^{1-\gamma}\right]}{\mathbb{E}\left[e^{-\rho_S}\left(\sum_{n\in N}\delta_{n,\eta}\right)^{1-\gamma}\right]}\left(\frac{\omega}{1-\omega}\right)^{-\gamma}\right]^{\frac{1}{\gamma}}+1\right]^{-1} \\
\theta_S &= \left[\frac{\left(\frac{e^{-\rho_H}}{e^{-\rho_S}}\right)^{\frac{1}{\gamma}}(1-\omega)+\omega}{\omega}\right]^{-1} = \frac{\omega}{\left(\frac{e^{-\rho_H}}{e^{-\rho_S}}\right)^{\frac{1}{\gamma}}(1-\omega)+\omega} \qquad \text{(C.2)}
\end{aligned}
$$

Substituting the solution for θ_S into the second equation of C.1 verifies the guess of $\theta = 0$. Notice that the sharing rule is time-invariant, current and future dividends play no role, it is purely a function of the state variable ω and basic parameters. The threshold when the capital constraint 2.4 starts binding occurs when $\omega < \tilde{\omega}$ that is identified at

$$
\beta = \frac{\tilde{\omega}}{\left(\frac{e^{-\rho_H}}{e^{-\rho_S}}\right)^{\frac{1}{\gamma}}(1-\tilde{\omega})+\tilde{\omega}} \rightarrow \tilde{\omega} = \frac{\left(\frac{e^{-\rho_H}}{e^{-\rho_S}}\right)^{\frac{1}{\gamma}}\beta}{\left(1+\beta\left(\left(\frac{e^{-\rho_H}}{e^{-\rho_S}}\right)^{\frac{1}{\gamma}}-1\right)\right)} \qquad \text{(C.3)}
$$

C.2. **Case** $\{F > 0,\, \theta_S = \beta\}$**.** Use the first equation of C.1 to solve for $\omega(\theta)$ which then implicitly defines $\theta(\omega)$.

$$
\begin{aligned}
\mathbb{E}\left[e^{-\rho_S}\left(\frac{\omega\left(\sum_{n\in N}\delta_n\right)}{\beta\left(\sum_{n\in N}\delta_{n,\eta}\right)-\theta}\right)^{\gamma}\right] &= \mathbb{E}\left[e^{-\rho_H}\left(\frac{(1-\omega)\left(\sum_{n\in N}\delta_n\right)}{(1-\beta)\left(\sum_{n\in N}\delta_{n,\eta}\right)+\theta}\right)^{\gamma}\right] \\
\left(\frac{1-\omega}{\omega}\right) &= \left(\frac{\mathbb{E}\left[e^{-\rho_S}\left(\frac{1}{\beta\left(\sum_{n\in N}\delta_{n,\eta}\right)-\theta}\right)^{\gamma}\right]}{\mathbb{E}\left[e^{-\rho_H}\left(\frac{1}{(1-\beta)\left(\sum_{n\in N}\delta_{n,\eta}\right)+\theta}\right)^{\gamma}\right]}\right)^{\frac{1}{\gamma}} \\
\omega(\theta) &= \left(\left(\frac{\mathbb{E}\left[e^{-\rho_S}\left(\frac{1}{\beta\left(\sum_{n\in N}\delta_{n,\eta}\right)-\theta}\right)^{\gamma}\right]}{\mathbb{E}\left[e^{-\rho_H}\left(\frac{1}{(1-\beta)\left(\sum_{n\in N}\delta_{n,\eta}\right)+\theta}\right)^{\gamma}\right]}\right)^{\frac{1}{\gamma}}+1\right)^{-1} \qquad \text{(C.4)}
\end{aligned}
$$

An explicit solution for $\theta(\omega)$ is not immediate, so I present the solution as an implicit function. Over the domain $[0, \tilde{\omega}]$, $\theta(\omega)$ is monotonic and decreasing in ω and can be computed as the inverse function of $\omega(\theta)$. This inverse is unique over the domain $\theta \in \left(0, Min_{\eta}\left[\beta\left(\sum_{n\in N}\delta_{n,\eta}\right)\right]\right)$. This condition ensures positive terminal consumption. The function is uniformly decreasing in the state variable ω.

The transfer F is then directly defined as $F(\theta)$ from the second equation of C.1 with $\theta(\omega)$ implicitly defined in equation C.4 as the wedge in the market prices of risk between households and specialists.

$$
\begin{aligned}
F(\theta(\omega)) &= \mathbb{E}\left[e^{-\rho_H}\left(\frac{(1-\omega)\left(\sum_{n\in N}\delta_n\right)}{(1-\beta)\left(\sum_{n\in N}\delta_{n,\eta}\right)+\theta}\right)^{\gamma}\left(\sum_{n\in N}\delta_{n,\eta}\right)\right] \\
&\quad -\mathbb{E}\left[e^{-\rho_S}\left(\frac{\omega\left(\sum_{n\in N}\delta_n\right)}{\beta\left(\sum_{n\in N}\delta_{n,\eta}\right)-\theta}\right)^{\gamma}\left(\sum_{n\in N}\delta_{n,\eta}\right)\right] \qquad \text{(C.5)}
\end{aligned}
$$

C.3. **Prices.** Using the results of the previous two subsection, prices are readily computed. Compute the risk-free rate

$$
\begin{aligned}
1+rf_{T-1}^{\theta_S>\beta}(\omega) &= \left(\mathbb{E}\left[\frac{U'(c_{S,T})}{U'(c_{S,T-1})}\right]\right)^{-1} = \left(\mathbb{E}\left[e^{-\rho_S}\left(\frac{\omega\left(\sum_{n\in N}\delta_n\right)}{\frac{\omega}{\left(\frac{e^{-\rho_H}}{e^{-\rho_S}}\right)^{\frac{1}{\gamma}}(1-\omega)+\omega}\left(\sum_{n\in N}\delta_{n,\eta}\right)}\right)^{\gamma}\right]\right)^{-1} \\
&= \left(e^{-\rho_S}\left(\sum_{n\in N}\delta_n\right)^{\gamma}\left(\left(\frac{e^{-\rho_H}}{e^{-\rho_S}}\right)^{\frac{1}{\gamma}}(1-\omega)+\omega\right)^{\gamma}\mathbb{E}\left[\left(\sum_{n\in N}\delta_{n,\eta}\right)^{-\gamma}\right]\right)^{-1} \qquad \text{(C.6)} \\
1+rf_{T-1}^{\theta_S=\beta}(\theta(\omega)) &= \left(e^{-\rho_S}\left(\sum_{n\in N}\delta_n\right)^{\gamma}\omega^{\gamma}\mathbb{E}\left[\left(\beta\left(\sum_{n\in N}\delta_{n,\eta}\right)-\theta\right)^{-\gamma}\right]\right)^{-1}
\end{aligned}
$$

as well as the risk premia for risky asset $n \in N$

$$
\begin{aligned}
1+\mu_{n,T-1}^{\theta_S>\beta}(\omega) &= \left(\frac{\mathbb{E}\left[\delta_{n,\eta}\right]}{P_{n,T-1}^{\theta_S>\beta}}/\left(1+rf_{T-1}^{\theta_S>\beta}\right)\right)=\frac{\mathbb{E}\left[\delta_{n,\eta}\right]\mathbb{E}\left[\left(\sum_{n\in N}\delta_{n,\eta}\right)^{-\gamma}\right]}{\mathbb{E}\left[\left(\sum_{n\in N}\delta_{n,\eta}\right)^{-\gamma}\delta_{n,\eta}\right]} \\
&= 1-\frac{COV\left(\delta_{n,\eta},\left(\sum_{n\in N}\delta_{n,\eta}\right)^{-\gamma}\right)}{\mathbb{E}\left[\left(\sum_{n\in N}\delta_{n,\eta}\right)^{-\gamma}\delta_{n,\eta}\right]} \qquad \text{(C.7)} \\
1+\mu_{n,T-1}^{\theta_S=\beta}(\omega) &= \frac{\mathbb{E}\left[\delta_{n,\eta}\right]\mathbb{E}\left[\left(\beta\left(\sum_{n\in N}\delta_{n,\eta}\right)-\theta\right)^{-\gamma}\right]}{\mathbb{E}\left[\left(\beta\left(\sum_{n\in N}\delta_{n,\eta}\right)-\theta\right)^{-\gamma}\delta_{n,\eta}\right]} \\
&= 1-\frac{COV\left(\delta_{n,\eta},\left(\beta\left(\sum_{n\in N}\delta_{n,\eta}\right)-\theta\right)^{-\gamma}\right)}{\mathbb{E}\left[\left(\beta\left(\sum_{n\in N}\delta_{n,\eta}\right)-\theta\right)^{-\gamma}\delta_{n,\eta}\right]}
\end{aligned}
$$

Appendix D. Analytical Solution when $\hat{N}=\{1\}$

Households enjoy access to risky asset 1. The results proceed as in the previous section. One additional equation is added to the system, reflecting the choice of households in direct exposure to asset 1. Denote the portfolio choice variable with respect to asset 1 α for the specialist and, invoking market clearing, $1-\alpha$ for households.

$$
\begin{aligned}
\mathbb{E}\left[e^{-\rho_S}\left(\frac{\omega\left(\sum_{n\in N}\delta_n\right)}{\theta_S\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)-\theta}\right)^{\gamma}\right] &= \\
\mathbb{E}\left[e^{-\rho_H}\left(\frac{(1-\omega)\left(\sum_{n\in N}\delta_n\right)}{(1-\theta_S)\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)+(1-\alpha)\delta_{1,\eta}+\theta}\right)^{\gamma}\right] & \\
\mathbb{E}\left[e^{-\rho_S}\left(\frac{\omega\left(\sum_{n\in N}\delta_n\right)}{\theta_S\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)-\theta}\right)^{\gamma}\delta_{1,\eta}\right] &= \\
\mathbb{E}\left[e^{-\rho_H}\left(\frac{(1-\omega)\left(\sum_{n\in N}\delta_n\right)}{(1-\theta_S)\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)+(1-\alpha)\delta_{1,\eta}+\theta}\right)^{\gamma}\delta_{1,\eta}\right] & \\
\mathbb{E}\left[e^{-\rho_S}\left(\frac{\omega\left(\sum_{n\in N}\delta_n\right)}{\theta_S\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)-\theta}\right)^{\gamma}\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)\right] &= \\
\mathbb{E}\left[e^{-\rho_H}\left(\frac{(1-\omega)\left(\sum_{n\in N}\delta_n\right)}{(1-\theta_S)\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)+(1-\alpha)\delta_{1,\eta}+\theta}\right)^{\gamma}\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)\right]-F & \\
F(\beta-\theta_S) &= 0 \qquad \text{(D.1)} \\
\theta_S &\geq \beta
\end{aligned}
$$

D.1. **Case $\{F=0,\,\theta_S>\beta\}$.** Guessing $\theta=0$ and $\alpha=1$ delivers the same sharing rule as in equation C.2 with the threshold C.3. In other words, the two economies are identical for as long as the capital constraint 2.4 is slack.

D.2. **Case** $\{F > 0,\, \theta_S = \beta\}$**.** Take the first two expectational equations from system D.1 and solve both for $\omega(\theta, \alpha)$.

$$\omega(\theta,\alpha) = \left(\left(\frac{\mathbb{E}\left[e^{-\rho_S}\left(\frac{1}{\beta\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)-\theta}\right)^{\gamma}\right]}{\mathbb{E}\left[e^{-\rho_H}\left(\frac{1}{(1-\beta)\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)+(1-\alpha)\delta_{1,\eta}+\theta}\right)^{\gamma}\right]}\right)^{\frac{1}{\gamma}}+1\right)^{-1}$$

$$\omega(\theta,\alpha) = \left(\left(\frac{\mathbb{E}\left[e^{-\rho_S}\left(\frac{1}{\beta\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)-\theta}\right)^{\gamma}\delta_{1,\eta}\right]}{\mathbb{E}\left[e^{-\rho_H}\left(\frac{1}{(1-\beta)\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)+(1-\alpha)\delta_{1,\eta}+\theta}\right)^{\gamma}\delta_{1,\eta}\right]}\right)^{\frac{1}{\gamma}}+1\right)^{-1} \tag{D.2}$$

The unknowns α and θ are again implicitly defined as functions of the state variable ω. The inverse is unique over the domain that guarantees positive specialists consumption in all future states. The transfer F is again directly defined from the third expectational equation as

$$\begin{aligned} &F(\theta(\omega),\,\alpha(\omega)) = \\ &\mathbb{E}\left[e^{-\rho_H}\left(\frac{(1-\omega)\left(\sum_{n\in N}\delta_n\right)}{(1-\theta_S)\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)+(1-\alpha)\delta_{1,\eta}+\theta}\right)^{\gamma}\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)\right] \\ &-\mathbb{E}\left[e^{-\rho_S}\left(\frac{\omega\left(\sum_{n\in N}\delta_n\right)}{\theta_S\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)-\theta}\right)^{\gamma}\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)\right] \end{aligned} \tag{D.3}$$

D.3. **Prices.** Compute the risk-free rate

$$\begin{aligned} 1+rf_{T-1}^{\theta_S>\beta}(\omega) &= \left(e^{-\rho_S}\left(\sum_{n\in N}\delta_n\right)^{\gamma}\left(\left(\frac{e^{-\rho_H}}{e^{-\rho_S}}\right)^{\frac{1}{\gamma}}(1-\omega)+\omega\right)^{\gamma}\mathbb{E}\left[\left(\sum_{n\in N}\delta_{n,\eta}\right)^{-\gamma}\right]\right)^{-1} \\ 1+rf_{T-1}^{\theta_S=\beta}(\theta(\omega)) &= \left(e^{-\rho_S}\left(\sum_{n\in N}\delta_n\right)^{\gamma}\omega^{\gamma}\mathbb{E}\left[\left(\beta\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)-\theta\right)^{-\gamma}\right]\right)^{-1} \end{aligned} \tag{D.4}$$

as well as the risk premia for risky asset $n \in N$

$$\begin{aligned} 1+\mu_{n,T-1}^{\theta_S>\beta}(\omega) &= \left(\frac{\mathbb{E}[\delta_{n,\eta}]}{P_{n,T-1}^{\theta_S>\beta}}\Big/\left(1+rf_{T-1}^{\theta_S>\beta}\right)\right) = \frac{\mathbb{E}[\delta_{n,\eta}]\,\mathbb{E}\left[\left(\sum_{n\in N}\delta_{n,\eta}\right)^{-\gamma}\right]}{\mathbb{E}\left[\left(\sum_{n\in N}\delta_{n,\eta}\right)^{-\gamma}\delta_{n,\eta}\right]} \\ &= 1-\frac{COV\left(\delta_{n,\eta},\left(\sum_{n\in N}\delta_{n,\eta}\right)^{-\gamma}\right)}{\mathbb{E}\left[\left(\sum_{n\in N}\delta_{n,\eta}\right)^{-\gamma}\delta_{n,\eta}\right]} \\ 1+\mu_{n,T-1}^{\theta_S=\beta}(\omega) &= \frac{\mathbb{E}[\delta_{n,\eta}]\,\mathbb{E}\left[\left(\beta\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)-\theta\right)^{-\gamma}\right]}{\mathbb{E}\left[\left(\beta\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)-\theta\right)^{-\gamma}\delta_{n,\eta}\right]} \\ &= 1-\frac{COV\left(\delta_{n,\eta},\left(\beta\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)-\theta\right)^{-\gamma}\right)}{\mathbb{E}\left[\left(\beta\left(\alpha\delta_{1,\eta}+\sum_{n\in N\setminus\{1\}}\delta_{n,\eta}\right)-\theta\right)^{-\gamma}\delta_{n,\eta}\right]} \end{aligned} \tag{D.5}$$

Appendix E. Lagrangian for Investor i_k

$$
\begin{aligned}
\mathbb{L}_{t,k} \quad = \quad & \underset{c_{k,t},\theta_{k,I,k_t},\left\{\alpha_{n^k}\right\},\left\{\theta_{k',I^{k'},t}\right\},\theta_{k,B,t}}{\overset{(E.1)}{max}} \mathbb{E}_t\left[\sum_{\tau=0}^{T-t} U_k\left(c_{k,t+\tau}\right)\right] \\
& -\mathbb{E}_t\left[\sum_{\tau=0}^{T-t}\phi_{k,t+\tau}\left(c_{t+\tau,k}+\theta_{k,I^k,t+\tau}\left(\sum_{n^k=1}^{N^k}\alpha_{n^k,t+\tau}P_{n^k,t+\tau}\right)\right.\right. \\
& +\sum_{k'\in K'}\left(1-\sum_{k'\in K'}\theta_{k',I^{k'},t+\tau}\right)\left(\sum_{n^{k'}=1}^{N^{k'}}\alpha_{n^{k'},t+\tau}P_{n^{k'},t+\tau}+F_{k,I^{k'},t+\tau}\right) \\
& -\sum_{k'\in K'}\left(1-\sum_{k'\in K'}\theta_{k',I^k,t+\tau}\right)F_{k',I^k,,t+\tau}+\theta_{k,B,t+\tau}B_{t+\tau}-\left(\theta_{k,I^k,t+\tau-1}\left(\sum_{n^k=1}^{N^k}\alpha_{n^k,t+\tau-1}\left(P_{n^k,t+\tau}+\delta_{n^k,t+\tau}\right)\right)\right. \\
& \left.\left.\left.+\sum_{k'\in K'}\left(1-\sum_{k'\in K'}\theta_{k',I^{k'},t+\tau-1}\right)\left(\sum_{n^{k'}=1}^{N^{k'}}\alpha_{n^{k'},t+\tau-1}\left(P_{n^{k'},t+\tau}+\delta_{n^{k'},t+\tau}\right)\right)+\theta_{k,B,t+\tau-1}\right)\right)\right] \\
& +\mathbb{E}_t\left[\sum_{\tau=0}^{T-t}\sum_{k'\in K'}\phi^{CC}_{k,k',t+\tau}\left(\beta-\theta_{k',I^{k'},t+\tau}\right)\right] \\
& \theta_{k',I^{k'},t+\tau}\geq\beta\,\forall k'\in K'
\end{aligned}
$$